# Foreign Direct Investment Cycles and Intellectual Property Rights in Developing Countries

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**Abstract**

This paper develops a quality ladder model in which the technology gap between the North and the South is endogenously determined. Foreign direct investments (FDI) occur cyclically: New FDI arrives if and only if the technology gap reaches some threshold. A stronger intellectual property rights (IPR) in the South discourages imitation and reduces the FDI cycle length. A smaller market size and more imitating firms in the South tend to enlarge the FDI cycle length. The social welfare of the South is decreasing in the FDI cycle length, but is decresing in IPR strength given cycle length. The optimal IPR strength balances these two effects, and it is non-monotonic in the market size and increasing in the number of imitating firms.

**JEL Classification**: F21; O31; O34

**Key Words**: Intellectual Property Rights; Foreign Direct Investment; Innovation; Imitation; Product Cycles

## 1 Introduction

There has been an ongoing debate as to whether developing countries should strengthen their intellectual property rights (IPR). Opponents argue that strong IPR regimes reduce consumer welfare by prolonging innovators' monopoly power, and slow down the technology progress of developing countries by discouraging imitation. Proponents counteract with the argument that strong IPR regimes encourage foreign direct investment (FDI) from developed countries (North) to developing countries (South), which benefits the South.

Based on the variety expansion model of Grossman and Helpman (1991b), Helpman (1993) shows that strong IPR in the South reduces the rate of imitation and the technology progress in the South, shifts production to the North, and slows down innovations in the North as more resources are devoted to production. Lai (1998) introduces FDI into Helpman's model and reverses some of Helpman's predictions. Strong IPR in the South encourages FDI, which shifts production to the South and speeds up innovations in the North. In both papers, imitation is exogenous and costless: stronger IPR is assumed to be equivalent to a decrease in

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imitation intensity.[1] Based on the quality ladder model of Grossman and Helpman (1991a), Glass and Saggi (2002) demonstrate that strong IPR in the South discourages FDI and the rate of innovation in the North falls. They treat imitation intensity as endogenous and a stronger IPR is equivalent to an increase in imitation cost. As the South strengthens its IPR, the cost of imitation increases. As a result, more Southern resources are devoted to inefficient imitation, which crowds out FDI through the South resource constraint. Glass and Wu (2007) adopt the quality ladder model, and treat imitation as exogenous and costless, as in Lai (1998). And they find that stronger IPR in the South discourages FDI and slows down innovation in the North. The key effect they identified is the adjustment of the relative wage between the North and the South.

Unlike the aforementioned papers, which focus on the impacts of IPR in the South on global innovation rate, this paper evaluates IPR in the South mainly from the South perspective. Based on the quality ladder model of Grossman and Helpman (1991a), we develop a dynamic model investigating the impacts of South IPR on the frequency of FDI, imitation, and the technology progress of the South. The distinguishing feature of this paper is that we endogenize the technology gap between the South and the North. Unlike the existing quality ladder models in which the South can only be one or two steps, which is exogenously given, behind the North on the quality ladder, in our model the South could be $n$ steps behind, and the lagging steps are endogenously determined. In particular, our model generates FDI cycles: new FDI, which brings new generations of products, occurs only when the technology gap between the South and the North exactly reaches some threshold. The FDI cycle length captures how frequent new FDI occurs, and in general determines the technology gap between the North and the South. Within this framework, we identify the short run effect and the long run effect of South IPR. In the short run (within each FDI cycle), a stronger IPR tends to discourage imitation and reduce South welfare. However, in the long run (across FDI cycles), a stronger IPR tends to reduce the FDI cycle length (FDI becomes more frequent) and increase South welfare. Our framework enables us to quantitatively evaluate the short run effect and the long run effect in the same model, and derives the optimal IPR which maximizes the South welfare.

Another feature of our model is that we provide a more detailed micro-foundation about how the equilibrium imitation intensity in the South is determined, and how it is affected by the IPR strength, while in the existing literature imitation in the South is typically modelled in a reduced way. This allows us to study the effects of the market size and the industry structure of the South on FDI, imitation, and the optimal IPR.[2]

The existence of FDI cycle means that the current technology leaders in the North will not always make FDI in the South.[3] Anecdotal evidence suggests that FDI cycle exists in some industries. For instance, Volkswagen Passat B2 was introduced in Europe in 1981. Its variant Santana has been produced in China since 1986, and another variant Quantum was produced in

[1] A recent paper by Branstetter and Saggi (2011) endogenizes both FDI and imitation.

[2] The expense of this is that our model is a partial equilibrium model, while all the previously mentioned papers are general equilibrium models, which allows them to capture the endogenous adjustment of wages and the tradeoff between production and R&D.

[3] Consider HD TV as an example, and suppose the North keeps innovating new generations of TVs with higher resolutions. An FDI cycle length of 3 means that the current technology leaders in the North will do FDI in every 3 generations, say generations of 3, 6, 9 and so forth. And the current technology leaders of other generations will not do FDI.

Brazil from 1985 to 2002. In late 1980s, new generations of Passat, B3 and B4, were introduced in Europe (1988) and North American (1990). But they were never produced in China, and Volkswagen started to produce them in South America only after 1995. However, shortly after the newer generation of Passat, B5, was introduced in 1996 in Europe, Volkswagen started to produce it in China.[4]

The main ingredients of our model are as follows. Innovations only occur in the North, and the rate of innovation is exogenous as our focus is on the South. There is no international trade, which means that in order to sell in the South, Northern firms have to make FDI. We assume that FDI is costly: each FDI has to incur some fixed (sunk) cost. There are a fixed number of firms in the South, who are active in imitation. The Southern firms can imitate the products of FDI, but they are not able to imitate the products of the Northern firms who only produce in the North. Imitation is costly and imitation is endogenous. FDI and Southern firms engage in price (Bertrand) competition. IPR strength is modeled as the probability that a successful imitation is ruled illegal. The timing is that the Southern government chooses and commits to an IPR strength in the very beginning.

In equilibrium FDI occurs cyclically. The underlying reason is that FDI entails a fixed cost. Due to Bertrand competition, the price that a new FDI charges is increasing in the technology gap between the current leader in the North and that in the South. If the technology gap is not big enough, then new FDI by the current leader in the North will not be profitable, as it can only charge a lower price and is not able to cover its fixed cost. Every new generation of FDI faces the threat of imitation. Once a Southern firm successfully imitates and is ruled legal, that firm replaces the FDI serving the Southern market as Southern firms have a lower production cost. The equilibrium intensity of imitation decreases over an FDI cycle. This is because as the technology frontier of the North advances, the next FDI, which will render the current FDI product profitless, becomes more imminent. As a result, the expected duration of monopoly of successful imitation decreases, and Southern firms' incentive to imitate decreases as well.

As the Southern IPR strength increases, the equilibrium FDI cycle length decreases or FDI occurs more frequently. A stronger IPR reduces the prospect of successful imitation thus reduces the equilibrium imitation intensity. This increases the profitability of FDI and reduces the equilibrium cycle length. Since a smaller cycle length means a smaller (average) technology gap between the North and the South, this result implies that a strong IPR tends to reduce the technology gap, and on average speed up the technology progress of the South.[5]

We also show that the equilibrium cycle length is decreasing in the size of the Southern market and increasing in the number of Southern firms. A big market size directly increases the profitability of FDI, but it also increases the profitability of imitation thus encourages imitation, and this indirect effect tends to reduce the profitability of FDI. It turns out that the direct effect dominates. As the number of Southern firms increases, each firm will increase its imitation intensity as now there is more competition for imitation. Thus the aggregate

[4]Source: http://en.wikipedia.org/wiki/Volkswagen_Passat
http://en.wikipedia.org/wiki/Volkswagen_Santana
http://en.wikipedia.org/wiki/Shanghai_Volkswagen_Automotive

[5]More details regarding the realtionship between the IPR strength and the technology gap can be found in subsection 4.4.

imitation intensity increases as well. This decreases the profitability of FDI and leads to an increase in the equilibrium cycle length.

We then study the optimal IPR strength that maximizes the discounted social welfare of the South. In particular, we identified two main effects. The first effect is the "free upgrade" effect. Across cycles, Southern consumers pay the same price for higher quality products. Thus each new generation of FDI brings free upgrade to Southern consumers. A smaller FDI cycle length brings more frequent (though of smaller magnitudes) upgrades to consumers, which due to discounting leads to a higher discounted consumer welfare. The second effect is the "imitation effect." Within each cycle, successful imitation always increases Southern welfare as it reduces the price consumers pay and brings profit to Southern firms. Given the cycle length, the Southern government tends to induce the highest possible aggregate imitation intensity. This is because the private benefit of imitation always falls short of the social benefit, as firms care only about profits and fail to internalize consumers' benefits from a reduced price.

The free upgrade effect implies that the IPR strength should be high in order to reduce the cycle length. However, the imitation effect implies that the IPR strength should be set as low as possible given any cycle length. The optimal IPR strength balances these two effects, and they imply that the optimal IPR strength will not be the zero strength or the full strength. Essentially, the imitation effect captures the short-run (within a cycle) benefit of lowering IPR strength as it encourages imitation, while the free upgrade effect reflects the long-run (across cycles) cost as it makes FDI less frequent and enlarges the technology gap between the North and the South.

We call the equilibrium cycle length under the optimal (full) IPR strength the optimal (minimum) cycle length. Our simulation shows that the optimal cycle length either coincides with or is very close to the minimum cycle length. This implies that quantitatively the free upgrade effect is the dominant effect. Several comparative statics results emerge from our analysis and simulation. First, the optimal cycle length is weakly decreasing in the Southern market size. As the market size increases, the optimal IPR strength decreases if the optimal cycle length remains the same, and it jumps up if the optimal cycle length decreases. Thus our model predicts a non-monotonic relationship between IPR and domestic market size. Second, the optimal cycle length is independent of the imitation costs, and the optimal IPR strength is decreasing in the imitation costs. Since imitation costs are typically decreasing in the level of economic development, this means that there is a positive correlation between IPR and GDP per capita. These two predictions are largely consistent with the empirical evidence. Finally, the optimal cycle length is almost independent of the number of Southern firms. As the number of Southern firms increases, the optimal IPR strength increases. Therefore, IPR is positively correlated with the competitiveness of the Southern industry.

TRIPS agreement in 1996 tried to impose a common framework for IPR globally. The results of this paper suggest that it is hard to set a uniform IPRs requirement across developing countries, as developing countries, in order to maximize their own welfare, have incentives to implement different IPR strength. Another result we found, which is surprising, is that the North does not always benefit from an increase in the Southern IPR. This is because an increase in the Southern IPR might reduce the equilibrium cycle length. In this case, FDI would occur more frequently, which reduces the expected length of the monopoly of any existing FDI, or the competition among different generations of the Northern FDIs is intensified. This effect tends to reduce the profit of Northern FDI and make the North worse off.

Based on the variety expansion model, Grossman and Lai (2004) study an IPR setting game played between the South and the North. They show that in equilibrium the North always sets a stronger IPR than the South. But in their model there is no FDI. In a different approach, Markusen (2001) develops an agency model and studies how IPR in the South affects Northern firms' incentive to make FDI and the Southern welfare.[6] His main result is that strong IPR is welfare improving if it induces Northern firms switching from exporting to FDI, and is welfare reducing if FDI occurs anyway. In a static model, Chen and Puttitanun (2005) study the optimal IPR from the South perspective. A strong IPR slows down the imitation of Northern products, but encourages domestic innovation. They predict that the optimal IPR of the South depends on the level of economic development in a non-monotonic way, first decreasing then increasing.[7] Auriol et al. (2012) focus on the following trade-off: a weak IPR in the South speeds up imitation, but prevents the South from legally exporting to the North. As a result, the South's incentive to implement a strong IPR depends on the size of its domestic market, with bigger developing counties having weaker incentives to set a high IPR strength.

Another related paper to ours is Glass and Saggi (1998), which develops a quality ladder model in which the North can be either two steps ahead over the South in some industries or only one step ahead. However, FDI can only be one step ahead of the South's indigenous technology level. In the former case, the quality of FDI is one step below the state-of-art technology in the North and is called low-quality FDI, while in the latter case FDI uses the state-of-art technology and is called high quality FDI. They study how the mix of high-quality and low-quality FDI depends on the imitation in the South and innovation in the North. In their model, FDI occurs at every quality level and IPR strength is not considered.

The rest of the paper is organized as follows. Section 2 sets up the model. In Section 3 we characterize the equilibrium FDI cycle and investigate how the cycle length depends on various factors. Section 4 studies the optimal IPR strength that maximizes the discounted social welfare, and how it is affected by the market size, imitation costs, and the number of firms in the South. Concluding remarks are offered in Section 5. All the proofs can be found in the Appendix.

## 2 The Model

We study an economy with two countries: the South and the North. There is a single industry in both countries, which produces a single good. The good can be of different qualities on a quality ladder, a la Grossman and Helpman (1991a). Denote the quality level of the good as $q_j \in \{0, 1, 2, 3, ...\}$. The quality improvement between any two adjacent quality levels is $\lambda > 1$. That is, if a consumer consumes a good of quality $q_j$, he derives a gross utility of $\lambda^{q_j}$. In each country, at any instant there exists a maximum technologically feasible quality level, which

[6] New FDI occurs exogenously in every two periods. And FDI needs to hire a local agent in both periods. The local agent hired in the first period might defect and start a rival firm in the second period.

[7] Zigic (2000), in a trade model, shows that the South might increase its IPR due to the strategic considerations of Northern firms. Yang and Maskus (2001) study how the IPR in the South affects Northern firms' incentive to innovate and to license advanced techonolgies.

can be improved through either innovation or imitation. Time is continuous, with each agent's discount rate being $\rho$.

Two countries differ in their innovating abilities. We assume that only the North innovates, while the South only imitates. The industry in the North is perfectly competitive. Denote the aggregate R&D intensity in the North as $\iota$, which is exogenously given. That is, at each instant with a Poisson arrival rate $\iota$ some North firm(s) improves the maximum quality level by one step. Note that Northern firms have to climb up the quality ladder step by step (no skipping). The quality level of the leading Northern firm is publicly observable. We do not model the Northern market in detail as our focus is on the Southern market. Northern firms might sell in the Southern market. We assume that there is no international trade. This means that, in order to sell in the Southern market, Northern firms have to make foreign direct investment (FDI).[8] Each FDI requires a sunk cost $F > 0$, and the marginal cost of production for each FDI in the South (regardless of quality) is $\xi \in (1, \lambda)$.

In the South, the industry has $N \geq 1$ firms. The Southern firms can improve quality level only through imitating products sold by the Northern FDI firm in the South, and they are not able to imitate the products produced only in the North.[9] This means that FDI is the only channel of international technology transfer, and without it the technology progress in the South will be stagnant. Denote the quality level of the up-to-date FDI as $q_{FDI}$, and the technology level of the leading Southern firm(s) as $q_S$. Let $\Delta \equiv q_{FDI} - q_S \in \{0, 1, 2, ...\}$ be the step difference between $q_{FDI}$ and $q_S$. By imitation, $q_S$ can jump to the level of $q_{FDI}$ directly, and this is the only source of technology progress in the South. Let $\mu_{Ij}$ be Southern firm $j$'s imitation intensity. With a Poisson arrival rate $\mu_{Ij}$ firm $j$ will successfully imitate the product of the FDI at each instant. Imitation is costly. By choosing imitation intensity $\mu_{Ij}$, at each instant $dt$ firm $j$ incurs a cost of $a_I(\Delta)\mu_{Ij}^2 dt$.[10] The cost $a_I(\Delta)$ is (weakly) increasing in the technological difference $\Delta$, which captures the fact that it is more difficult to imitate relatively more advanced technology.

There are a mass of $M_S$ consumers in the South. The marginal cost of production of any Southern firm is (normalized to) 1 regardless of the quality level. Note that Southern firms have advantages in production relative to Northern FDI firms, $\xi > 1$, which reflects the fact that FDI firms are operating in an unfamiliar environment. In the product market, firms compete in prices.

When a Northern firm with a quality level more advanced than any operating firms in the South makes FDI, it acquires a patent from the Southern government. The patent potentially prevents Southern firms from imitating products of the FDI. The patent law and enforcement in the South, however, is not perfect. In particular, the IPR strength in the South is captured by one parameter $p \in [0, 1]$. If a Southern firm successfully imitates the FDI's product, with probability $p$ the imitating product is ruled illegal and cannot be sold in the market, and with

[8] An alternative setting is that there is international trade, but the South imposes tariff. Now the gross benefit of FDI is the save of tariff, instead of the access of the Southern market in the basic model. Our main results still hold qualitatively in this alternative setting.

[9] FDI creates local knowledge spillovers, hence it is much easier for Southern firms to imitate. Glass and Saggi (2002) make a similar but weaker assumption: it is less costly to imitate an FDI's product than to imitate a Northern firm's product.

[10] The quadratic cost function is not essential to our qualitative results, but it does simplify our computation. Any increasing and convex cost function would work, at the expense of more complicated algebra.

probability $1 - p$ it is ruled legal and sold in the market.[11] The probability $p$ is i.i.d. across firms and across different tries of the same firm. In the case that a Southern firm's successful imitation is ruled illegal, the successfully imitated product is discarded, and this firm has to start imitation from scratch.[12] This simplifying assumption ensures that all Southern firms are symmetric in terms of imitating, regardless whether a firm has successfully come up with imitations but ruled illegal beforehand.

The timing is as follows. First, the Southern government chooses the IPR strength $p$ at time 0 and commits to it afterwards. Then, after observing $p$, all firms play their parts over time. At time 0, the North and the South both are at the lowest quality ladder $q_j = 0$.

# 3 Equilibrium FDI Cycles

## 3.1 Preliminary Analysis

By backward induction, we first analyze the game played among firms, given the IPR strength $p$ of the South. In this subsection, we will show that in equilibrium FDI occurs periodically, or equilibrium exhibits FDI cycles, with the cycle length indicating the frequency of FDI. In particular, new FDI occurs if and only if the technology gap between the South and the North reaches some threshold. Let this threshold level of technology gap be $\overline{\Delta}$ (the number of steps on the quality ladder). We call $\overline{\Delta}$ the length of FDI cycles (simply cycle length sometimes).

In any instant of time, in the South either a single firm or two firms have the leading technology (will be verified later). If a single firm has the leading technology, then it must be an FDI firm. If a Southern firm successfully imitates and is ruled legal, then the FDI and the Southern firm both have the leading technology. Suppose only an FDI has the leading technology. In this case, there is no successful imitation product on the market, thus the technology gap between the FDI and Southern firms is $\overline{\Delta}$. Since firms are engaging in Bertrand competition and $\xi < \lambda$, the FDI is a monopoly in the Southern market, and it charges a "limit" price $\lambda^{\overline{\Delta}}$. Now suppose one Southern firm successfully imitates the FDI's product and is ruled legal.[13] In this case, two firms' products have the same quality. But since the Southern firm has cost advantage, the Southern firm becomes a monopoly in the market and charges a "limit" price $\xi$. Once a Southern firm successfully imitates and is ruled legal, all other Southern firms will stop imitating. This is because even if another Southern firm successfully imitates and is ruled legal, due to Bertrand competition, it would have earned a zero profit as two Southern firms have the same quality level and the same production cost.

Now we show that at any instant only the leading firm in the North will possibly make FDI in the South and FDI exhibits cycles. Let $\Delta_{NS}$ be the quality difference between the leading Northern firm's quality level and the leading technology in the South (including the

[11] Our modeling of the IPR strength is rather abstract. For more details of patent law, eg. patent leading breadth, lagging breadth, and patentibility, see O'Donoghue (1998).

[12] One can think that each successful imitation produces a slightly different version. If one version happens to be ruled illegal, then this version will never be ruled legal in future tries.

[13] At each instant, the probability that more than one Southern firm come up with successful imitation is negligible, relative to the probability that a single Southern firm successfully imitates.

most recent FDI).[14] Recall that the Northern industry is perfectly competitive. This means that on the quality ladder different Northern firms will be the leading firm at different times. Moreover, only one firm will be the leading firm in the North. The leading firm in the North has an incentive to make FDI in the South, in order to earn extra profit. But with the sunk cost $F$, it might not be profitable due to the imitation threat of the Southern firms and the future FDI of the future Northern leading firms, which makes the current FDI obsolete. Therefore, new FDI will occur only if $\Delta_{NS}$ reaches some threshold level. Denote the expected gross discounted payoff of a new FDI as $V_F(\Delta_{NS})$. We argue that $\overline{\Delta}$ is the smallest $\Delta_{NS}$ such that $V_F(\Delta_{NS}) \geq F$. This is because once $\Delta_{NS}$ reaches $\overline{\Delta}$, new FDI becomes profitable and the current leading firm in the North will immediately carry out FDI. On the other hand, if the North-South technology gap is smaller than $\overline{\Delta}$, the current leading firm in the North will not carry out FDI as it is not profitable by the definition of $\overline{\Delta}$. Therefore, FDI must occur periodically or cyclically in equilibrium, with the cycle length being $\overline{\Delta}$.

To summarize, FDI occurs cyclically. Once the technology gap between the North and the South reaches $\overline{\Delta}$, the leading Northern firm at that moment immediately makes FDI in the South, which starts a new cycle. At that moment, the technology gap becomes 0. Within a cycle, the most recent FDI first has the monopoly in the Southern market, and Southern firms try to imitate the product of the FDI. Once imitation is successful and ruled legal, a Southern firm replaces the FDI as the monopoly of the Southern market. At the same time, the technology frontier in the North is advancing stochastically, which means that the technology gap is widening. Once the technology gap between the North and the South reaches $\overline{\Delta}$ again, the leading Northern firm at that moment immediately makes FDI, which ends the current cycle (regardless whether imitation has succeeded or not) and starts a new cycle. For simplicity, we assume that once a new FDI arrives in the South, the technology of the previous FDI becomes freely available to all Southern firms as it becomes obsolete.[15] This means that at the beginning of any cycle, all Southern firms have the same quality level, which is $\overline{\Delta}$ steps below the new FDI's.

### 3.2 Imitation

In this subsection we investigate Southern firms' incentive to imitate. Recall that $\Delta_{NS}$ is the quality difference between the leading Northern firm's quality level and the leading technology in the South. In particular, $\Delta_{NS} = \{0, 1, ..., \overline{\Delta} - 1\}$, which indicates the phases of the cycle. Note that $\Delta_{NS}$ affects Southern firms' incentive to imitate. This is because a bigger $\Delta_{NS}$ implying a shorter remaining length of the current cycle, as new FDI will arrive sooner in expectation, which will render successful imitation obsolete. Therefore, we need to treat $\Delta_{NS}$ as a state variable and keep track of it.

Denote $V_I(i, \overline{\Delta})$ as the discounted expected payoff of a successful imitator in the current cycle given $\Delta_{NS} = i$ and the cycle length being $\overline{\Delta}$, and $V_F(i, \overline{\Delta})$ as the discounted payoff of the most recent FDI. Let $\mu_{Ij}(i, \overline{\Delta})$ be firm $j$'s imitation intensity in state $i$. The aggregate

[14] The leading technology in the South is just the quality level of the most recent FDI. This is because successful imitation will not advance the quality level in the South beyond that of the most recent FDI.

[15] This is a standard assumption in the literature (see Glass and Saggi, 1998 and 2002), which makes the environment stationary.

imitation intensity is $\mu(i,\overline{\Delta}) = \sum_{j=1}^{N} \mu_{Ij}(i,\overline{\Delta})$. When there is no confusion, we sometimes suppress the argument of $\overline{\Delta}$ and simply write the endogenous variables as $V_I(i)$, $V_F(i)$, and $\mu(i)$, respectively. The value function of $V_I(i)$ can be written as:

$$\rho V_I(\overline{\Delta}-1) = M_S(\xi-1) + \iota[0 - V_I(\overline{\Delta}-1)],$$
$$\rho V_I(i) = M_S(\xi-1) + \iota[V_I(i+1) - V_I(i)] \text{ for } i \leq \overline{\Delta}-2.$$

The above value functions can be interpreted as follows. The flow payoff of holding the asset $V_I(i)$ equals to the instantaneous profit $M_S(\xi-1)$, plus the change in asset value: with probability $\iota$ the leading technology in the North advances by one step, hence $V_I(i)$ is changed to $V_I(i+1)$. Note that $V_I(\overline{\Delta}) = 0$, since when $\Delta_{NS} = \overline{\Delta}$ new FDI will arrive and the Southern firm loses the market. Solving the value functions recursively, we get

$$V_I(i) = M_S(\xi-1)\frac{1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}-i}}{\rho}. \tag{1}$$

Observing (1), we can see that $V_I(i)$ is decreasing in $i$. Intuitively, a bigger $i$ implies a shorter (expected) remaining length of the current cycle, which further implies a smaller value of successful imitation.

Now we derive Southern firms' equilibrium imitation intensities. We will focus on symmetric equilibrium in which all firms choose the same (individual) imitation intensity $\mu_I^*(i)$. Thus the equilibrium aggregate imitation intensity is $\mu^*(i) = N\mu_I^*(i)$. To derive the symmetric equilibrium, suppose all other Southern firms choose $\mu_I^*(i)$, and consider firm $j$. Firm $j$'s discounted payoff of imitation (before any successful imitation occurs), $w_j(\mu_{Ij}, i)$, can be written as (suppress argument $i$):

$$\rho w_j(\mu_{Ij}) = -a_I\mu_{Ij}^2 + \mu_{Ij}(1-p)(V_I - w_j) + (N-1)\mu_I^*(1-p)(0-w_j).$$

In the above expression, $-a_I\mu_{Ij}^2$ is the instantaneous payoff. With probability $\mu_{Ij}(1-p)$ firm $j$ successfully imitates and its product is ruled legal, in which case firm $j$ collects $V_I$. With probability $(N-1)\mu_I^*(1-p)$, one of the other firms successfully imitates and its product is ruled legal, in which case firm $j$'s payoff becomes 0.

Solving for $w_j(\mu_{Ij})$ from the above expression, we get

$$w_j(\mu_{Ij}) = \frac{\mu_{Ij}(1-p)V_I - a_I\mu_{Ij}^2}{\rho + (N-1)\mu_I^*(1-p) + \mu_{Ij}(1-p)}. \tag{2}$$

Firm $j$ chooses $\mu_{Ij}$ to maximize $w_j$. Taking derivative, we get

$$\frac{\partial w_j}{\partial \mu_{Ij}} \propto [(1-p)V_I - 2a_I\mu_{Ij}][\rho + (N-1)\mu_I^*(1-p)] - a_I(1-p)\mu_{Ij}^2.$$

Since the above expression is strictly decreasing in $\mu_{Ij}$, the FOC is necessary and sufficient in characterizing the equilibrium $\mu_{Ij}^*$. After imposing symmetry, $\mu_{Ij}^* = \mu_I^*$, the equation characterizing $\mu_I^*$ becomes

$$a_I(2N-1)\mu_I^{*2} + [2a_I\frac{\rho}{1-p} - (N-1)(1-p)V_I]\mu_I^* - V_I\rho = 0. \tag{3}$$

**Lemma 1** *(i) There is a unique symmetric equilibrium, with* $\mu_I^* \in (\frac{(N-1)(1-p)V_I}{(2N-1)a_I}, \frac{(1-p)V_I}{2a_I})$.
*(ii) Both* $\mu_I^*$ *and* $\mu^*$ *are increasing in* $V_I$*, decreasing in* $p$ *and* $a_I$*, and increasing in* $N$.

Part (ii) of Lemma 1 shows that the equilibrium (both the individual and aggregate) imitation intensities are increasing in the value of imitation "prize" $V_I$, decreasing in the IPR strength $p$ and imitation cost $a_I$, and increasing in the number of Southern firms $N$. The first three properties are easy to understand. As to the last property, note that an increase in the number of firms means that, other things equal, it becomes more likely that one of the other firms will succeed first in imitation. This increases the effective discount rate for each individual firm (see equation (2)). As a result, each individual firm increases imitation intensity in order to speed up its own imitation. Another property worth mentioning is that, as the IPR strength $p$ increases, the equilibrium aggregate imitation cost $a_I(\mu^*)^2/N$ would decrease since $\mu^*$ decreases. This is different from Glass and Saggi (2002), in which an increase in IPR, though reduces equilibrium imitation intensity, leads to a higher aggregate imitation cost and more resources being devoted to imitation.

For simplicity, we write $\mu^*(i)$ simply as $\mu(i)$, with the understanding that it denotes equilibrium aggregate imitation intensity. Following part (ii) of Lemma 1, we can see that $\mu(i)$ is decreasing in $i$. That is, the intensity of imitation is monotonically decreasing over a cycle. This is because the prize of successful imitation, $V_I(i)$, is decreasing over a cycle.

### 3.3 Incentive to make FDI

Now we analyze leading Northern firms' incentives to make FDI. The value function $V_F(i)$ can be written as:

$$\begin{aligned}
\rho V_F(\overline{\Delta}-1) &= M_S(\lambda^{\overline{\Delta}} - \xi) + [-(1-p)\mu(\overline{\Delta}-1)V_F(\overline{\Delta}-1) - \iota V_F(\overline{\Delta}-1)] \\
\rho V_F(i) &= M_S(\lambda^{\overline{\Delta}} - \xi) + [-(1-p)\mu(i)V_F(i) + \iota(V_F(i+1) - V_F(i))] \text{ for } i \leq \overline{\Delta}-2 \quad (4)
\end{aligned}$$

The RHS of the above equations has two terms. The first term is the FDI's instantaneous profit, before any successful imitation occurs. The second term is the change in asset value. With intensity $(1-p)\mu(i)$ imitation is successful, and the value changes from $V_F(i)$ to 0; with intensity $\iota$ the leading technology in the North advances by one step, hence $V_F(i)$ is changed to $V_F(i+1)$. Solving the value functions (4) recursively, we get

$$V_F(i) = M_S(\lambda^{\overline{\Delta}} - \xi) \sum_{j=i}^{\overline{\Delta}-1} \frac{1}{\rho + (1-p)\mu(j) + \iota} \prod_{k=i}^{j-1} \frac{\iota}{\rho + (1-p)\mu(k) + \iota}, \quad (5)$$

$$V_F(0) = M_S(\lambda^{\overline{\Delta}} - \xi) \sum_{j=0}^{\overline{\Delta}-1} \frac{1}{\rho + (1-p)\mu(j) + \iota} \prod_{k=0}^{j-1} \frac{\iota}{\rho + (1-p)\mu(k) + \iota}. \quad (6)$$

Generally, whether $V_F(i)$ is decreasing in $i$ (or over the cycle) cannot be determined with certainty. This is because an increase in $\Delta_{NS}$ has two effects on $V_F(i)$. First, an increase in $\Delta_{NS}$ implies that the FDI's expected time length as a monopoly in the current cycle (if imitation is not successful) is reduced. This tends to reduce $V_F(i)$. Second, an increase in

$\Delta_{NS}$ reduces the imitation intensity $\mu(i)$ as mentioned earlier, which reduces the possibility of successful imitation and thus tends to increase $V_F(i)$. Depending on which effect is stronger, $V_F(i)$ could either decrease or increase over the cycle.

Observing (1), we see that $V_I$ has a lower bound $\underline{V}_I = V_I(\overline{\Delta} - 1) = M_S(\xi - 1)/(\rho + \iota)$ and an upper bound $\overline{V}_I = \lim_{\overline{\Delta}\to\infty} V_I(0, \overline{\Delta}) = M_S(\xi - 1)/\rho$. For analytical convenience, we assume the following conditions hold regarding the analytical results throughout the paper.

**Condition 1** *The following condition holds:*

$$\lambda\iota < \rho + \iota \Leftrightarrow (\lambda - 1)\iota < \rho.$$

**Condition 2** *The following condition holds*

$$\lambda\rho > \rho + \iota \Leftrightarrow (\lambda - 1)\rho > \iota.$$

Condition 1 basically says that the speed of technology progress ($\lambda\iota$) in the North is lower than the discount rate, which ensures that consumers' discounted utility will not explode but converge to some well defined limit. Condition 2 requires that $\lambda$ and $\rho$ are big enough relative to $\iota$. This condition ensures that the changes in imitation intensity over the cycle is relatively small, as will be shown later. Combining conditions 1 and 2, we have $\frac{\iota}{\rho} < \lambda - 1 < \frac{\rho}{\iota}$. Thus, $\rho$ should be relatively big, $\iota$ should be relatively small, and $\lambda$ should lie in between.

**Lemma 2** *(i) For all $\overline{\Delta}$, $\mu(0, \overline{\Delta}) \leq \lambda\mu(\overline{\Delta} - 1, \overline{\Delta})$. (ii) $V_F(0, \overline{\Delta})$ is strictly increasing in $\overline{\Delta}$.*

The intuition for Lemma 2 is as follows. As the cycle length increases from $\overline{\Delta}$ to $\overline{\Delta} + 1$, there are four effects on the leading Northern firms' incentive to make FDI, $V_F(0)$. First, it implies that each FDI can charge a higher price ($\lambda^{\overline{\Delta}+1}$ instead of $\lambda^{\overline{\Delta}}$) in the case of no successful imitation. Second, an increase in the cycle length increases the expected length of the monopoly of each FDI. Both effects tend to increase $V_F(0)$. Third, an increase in the cycle length also implies that successful imitator will now enjoy a longer expected length of monopoly as well, and this effect tends to increase the intensity of imitation and reduce $V_F(0)$. Finally, a bigger $\overline{\Delta}$ weakly increases the cost of imitation, which tends to reduce the intensity of imitation and increase $V_F(0)$. Condition 2 ensures that the third effect is weaker than the first two effects, so that $V_F(0)$ is strictly increasing in the cycle length. In particular, Condition 2 implies that, the increase in the intensity of imitation due to an increase in cycle length is small enough relative to the first effect. Note that even with the imitation cost $a_I$ being constant in the cycle length $\overline{\Delta}$, $V_F(0)$ could be strictly increasing in $\overline{\Delta}$.

### 3.4 Equilibrium cycle length

Let $\Lambda$ be the set of $\overline{\Delta}$s such that $V_F(0, \overline{\Delta}) \geq F$. That is, $\Lambda = \{\overline{\Delta}^E : V_F(0, \overline{\Delta}^E) \geq F\}$. As we argued earlier, the equilibrium cycle length $\overline{\Delta}^*$ is the smallest $\overline{\Delta}^E$ among all $\overline{\Delta}^E \in \Lambda$. Note that, if $\Lambda$ is nonempty, then $\overline{\Delta}^*$ is unique. Since, by part (ii) of Lemma 2, $V_F(0)$ is strictly increasing in the cycle length, the equilibrium cycle length satisfies the following conditions: $V_F(0, \overline{\Delta}^* - 1) < F$ and $V_F(0, \overline{\Delta}^*) \geq F$. Thus we have the following proposition.

**Proposition 1** *Suppose $\Lambda$ is nonempty. Given the South's IPR strength p, FDI occurs periodically or cyclically: new FDI occurs when the technology gap exactly reaches $\overline{\Delta}^*$. The equilibrium length of the FDI cycle is unique and satisfies: $V_F(0, \overline{\Delta}^* - 1) < F$ and $V_F(0, \overline{\Delta}^*) \geq F$.*

Note that Condition 2 is sufficient but not necessary for $V_F(0, \overline{\Delta})$ being strictly increasing in $\overline{\Delta}$ and the characterization of equilibrium cycle length $\overline{\Delta}^*$ in Proposition 1. For instance, for the characterization in Proposition 1 to be valid, Condition 2 can be weakened. In particular, part (i) of Lemma 2 does not need to hold for all $\overline{\Delta}$, but only for the relevant range of $\overline{\Delta}$ such that $\overline{\Delta} \leq \overline{\Delta}^*$.[16] In numerical simulations we do not impose Condition 2. Instead we only choose the parameter values such that $V_F(0, \overline{\Delta})$ is strictly increasing in $\overline{\Delta}$ for $\overline{\Delta} \leq \overline{\Delta}^*$.

One feature we want to emphasize is that most endogenous variables are isomorphic or follow the same path in every FDI cycle. Specifically, the price charged by every generation of FDI is always $\lambda^{\overline{\Delta}^*}$, and the price charged by each generation of successful imitator is always $\xi$. Moreover, the imitation intensities of the same phase (the same $\Delta_{NS}$) across different cycles are always the same. What is different across cycles is that technology advances, and hence consumers' gross utilities increase, across cycles. Thus the Southern consumers are also benefiting from the technology advancement of the North as well.

Now we conduct comparative statics, investigating how the equilibrium cycle length changes as exogenous parameter values change.

**Proposition 2** *The equilibrium FDI cycle length, $\overline{\Delta}^*$, is: (i) weakly decreasing in p, the IPR strength of the South, and weakly decreasing in $a_I$, the cost of imitation; (ii) weakly increasing in $\xi$, the cost of FDI production; (iii) weakly decreasing in $\lambda$, the size of each step in the quality ladder; (iv) weakly increasing in $N$, the number of Southern firms; (v) weakly decreasing in $M_S$, the size of the Southern market.*

The intuition for the results in Proposition 2 is as follows. When the IPR strength $p$ increases, the probability of successful imitation is directly reduced and the incentive of imitation is indirectly dampened. Both tend to increase the profitability of FDI and reduce the equilibrium cycle length. Similarly, an increase in the imitation cost $a_I$ reduces imitation intensity and weakly reduces the equilibrium cycle length. When the cost of FDI production $\xi$ increases, the instantaneous profit of FDI decreases, and the instantaneous profit of successful imitation increases, which increases the intensity of innovation. Both effects reduce the profitability of FDI, and thus the equilibrium cycle length will increase.

When the step size of the quality ladder $\lambda$ increases, it tends to increase the price charged by the FDI, while the incentive of imitation is not affected. As a result, the equilibrium cycle length will decrease. The comparative statics regarding the Northern innovation rate $\iota$ is ambiguous. As $\iota$ increases, for any given $\overline{\Delta}$ it reduces the expected time length of the monopoly of the existing FDI. But at the same time, it reduces the expected time length of the monopoly of the successful imitator as well, which dampens the incentive to imitate and increases the value of FDI. Either effect could dominate. One can think that the step size of the quality ladder $\lambda$ depends on the patent policy in the North. In particular, a more stringent

[16] Specifically, part (i) of Lemma 2 only requires $\mu(0, \overline{\Delta}^*) \leq \lambda\mu(\overline{\Delta}^* - 1, \overline{\Delta}^*)$, which requires a weaker condition than Condition 2 as $V_I(0, \overline{\Delta}^*) < \lim_{\overline{\Delta} \to \infty} V_I(0, \overline{\Delta})$.

patentability requirement of the North implies a bigger $\lambda$. Thus part (iii) of Proposition 2 implies that a more stringent patentability requirement in the North leads to a smaller FDI cycle length, or FDI will occur more frequently.[17]

As the number of Southern firms $N$ increases, the aggregate imitation intensity increases, which reduces the profitability of FDI and increases the equilibrium cycle length. This implies that if the Southern industry is more competitive, then the FDI cycle will be longer or FDI occurs less frequently.

As the size of the Southern market $M_S$ increases, it increases the instantaneous profit of existing FDI. But at the same time, it increases the instantaneous profit of the successful imitator as well, which increases imitation intensity and reduces the value of FDI. However, the first effect dominates and the equilibrium cycle length will weakly decrease. Intuitively, FDI benefits more from an increase in the market size than successful imitators. This is because the profit margin of the FDI is $\lambda^{\overline{\Delta}} - \xi$, which is bigger than $\xi - 1$, the profit margin of successful imitators. Moreover, given Conditions 1 and 2, the changes in equilibrium imitation intensities are not that sensitive to changes in the prize of successful imitation $V_I$. The implication of this result is that, other things equal, a bigger domestic market will lead to a shorter FDI cycle length. More specifically, FDI should occur more frequently in developing countries with bigger markets, such as China, than in developing countries with smaller markets, such as Thailand.

Our model can be easily extended to the case with many symmetric industries. Although all the industries have the same equilibrium cycle length, the stochastic nature of innovation means that in real physical time the phases of cycles of different industries are in general staggered. Thus, the FDI cycle length in our model can be interpreted as the volume of FDI: a longer cycle length means a smaller volume. With this interpretation, previous results imply that developing countries with a bigger market size should have a bigger FDI volume. Moreover, Southern countries with stronger IPR should attract more FDI, which is supported by empirical evidence.[18]

By part (i) of Proposition 2, the equilibrium cycle length, $\overline{\Delta}^*$, is weakly decreasing in $p$. Note that $p \in [0, 1]$. Correspondingly, fixing other parameter values, the equilibrium cycle length has a lower bound and an upper bound, which we call the minimum cycle length ($p = 1$) and maximum cycle length ($p = 0$), and are denoted as $\overline{\Delta}^*_{\min}$ and $\overline{\Delta}^*_{\max}$, respectively. Specifically, when $p = 1$, $\mu(\cdot) = 0$, and $V_F(0, 1, \overline{\Delta})$ (where the second argument denotes $p = 1$) becomes

$$V_F(0, 1, \overline{\Delta}) = \frac{M_S(\lambda^{\overline{\Delta}} - \xi)}{\rho}[1 - (\frac{\iota}{\rho + \iota})^{\overline{\Delta}}].$$

The minimum cycle length $\overline{\Delta}^*_{\min}$ satisfies $V_F(0, 1, \overline{\Delta}^*_{\min}) \geq F$ and $V_F(0, 1, \overline{\Delta}^*_{\min} - 1) < F$. Similarly, the maximum cycle length $\overline{\Delta}^*_{\max}$ satisfies $V_F(0, 0, \overline{\Delta}^*_{\max}) \geq F$ and $V_F(0, 0, \overline{\Delta}^*_{\max} - 1) < F$. Note that $\overline{\Delta}^*_{\max}$ is bounded, since $\mu(\cdot)$ is bounded even if $p = 0$ as imitation is costly. To

[17] This implication should be viewed with caution. Given our assumption that the innovation rate in the North is exogenous, roughly $\lambda\iota$ is constant. As $\lambda$ increases, $\iota$ will decrease correspondingly. But given that the impact of changes in $\iota$ on the equilibrium cycle length is ambiguous, a more stringent patentability requirement in the North will likely reduce the equilibrium cycle length.

[18] Lee and Mansfield (1996) show that developing countries with stronger IPR attract more FDI from the U.S.. According to Park and Lippoldt (2003), IPR matters importantly to FDI but only modestly to trade. Awokuse and Yin (2010) find that the strengthening of IPR in China had positive and significant effects on FDI.

summarize, the number of possible equilibrium cycle length is finite: $\overline{\Delta}^* \in \{\overline{\Delta}^*_{\min}, \overline{\Delta}^*_{\min} + 1, ..., \overline{\Delta}^*_{\max}\} \equiv \Gamma$.

# 4 The Optimal IPR Strength

Analysis in the previous section shows that, given the IPR strength of the South, $p$, the pattern of equilibrium FDI cycles is uniquely determined. In this section, we focus on the first stage and study the optimal IPR strength from the Southern government's perspective. Specifically, the government tries to maximize its discounted social welfare.

## 4.1 Discounted social welfare

Let $CS_t$ and $PS_t$ be the consumer surplus and producer surplus of the South at time $t$. The Southern government's social welfare at time $t$, $w_t$, is the sum of $CS_t$ and $PS_t$. In particular, $w_t = CS_t + PS_t$. Let $W(p)$ be the (expected) discounted social welfare of the South in equilibrium, given $p$, evaluated at the very beginning of the time, time 0. We sometimes write $W(p)$ simply as $W$ whenever there is no confusion. Let $W_k(0)$ be the equilibrium discounted social welfare, starting in $k$th cycle with state $i = 0$. Note that $W = (\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*} W_1(0)$, since it takes $\overline{\Delta}^*$ steps for the first FDI to occur and the first FDI cycle to start. Denote $w_k(i, J)$ as the instantaneous social welfare in the $k$th cycle with state $i$, where $J = F, S$. In particular, $F$ stands for the case that an FDI is serving the market (no successful imitation) and $S$ stands for the case that a Southern firm is serving the market (after successful imitation). More explicitly

$$\begin{aligned} w_k(i, F) &= (M_s \lambda^{k\overline{\Delta}^*}) + [-M_s \lambda^{\overline{\Delta}^*} - a_I(\overline{\Delta}^*)\mu^2(i)/N], \\ w_k(i, S) &= (M_s \lambda^{k\overline{\Delta}^*}) + [-M_s]. \end{aligned}$$

To understand the above expressions, note that when an FDI is serving the market, the consumers get a flow (gross) utility of $M_s\lambda^{k\overline{\Delta}^*}$, and they pay a total price of $M_s\lambda^{\overline{\Delta}^*}$, and the Southern firms in total incur a flow imitation cost of $a_I(\overline{\Delta}^*)\mu^2(i)/N$. When a Southern firm is serving the market, again the consumers get a flow (gross) utility of $M_s\lambda^{k\overline{\Delta}^*}$ and pay a total price of $M_s\xi$. The Southern firms in total have a profit of $M_s(\xi - 1)$, and they no longer incur imitation costs.

Observing the two above expressions, we see that the first term $M_s\lambda^{k\overline{\Delta}^*}$ (consumers' gross utility) is increasing across cycles, but remains constant within cycles. The second terms are changing within cycles (depending on state $i$), but are isomorphic across cycles. Based on these observations, we can decompose $W_1(0)$, which is a discounted sum of instantaneous social welfare, into two components. The first one is the discounted sum of consumers' gross utility, which is growing due to periodic free upgrades across cycles. We denote this term (starting at the beginning of $k$th cycle) as $C_k$. The second one is the discounted sum of the remaining terms (could be interpreted as changes in social welfare due to imitation within cycles). This term is denoted as $R(i, J)$, where $J = F, S$. Recall that this term is stationary across cycles. In short, $W_1(0) = C_1 + R(0, F)$ and $W_k(0) = C_k + R(0, F)$, since every cycle starts with a new FDI who initially serves the Southern market.

We first calculate $C_1$. In particular,

$$C_k = \frac{M_s \lambda^{k\overline{\Delta}^*}}{\rho}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}] + (\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*} C_{k+1}.$$

In the above expression, the first term is the discounted consumer gross utility within the current cycle, while the second term is the continuation payoff. Solving recursively, we get

$$C_1 = \frac{M_s \lambda^{\overline{\Delta}^*}}{\rho} \frac{1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}}{1-(\frac{\lambda\iota}{\rho+\iota})^{\overline{\Delta}^*}}. \tag{7}$$

By equation (7), $C_1$ is bounded due to Condition 1, $\lambda\iota < \rho+\iota$, which essentially means that the rate of upgrade is less than the discount rate. One can think of $C_1$ as the increase in Southern welfare due to periodic free upgrades of products resulting from technology growth in the North. In other words, $C_1$ captures the trend of growth across cycles. Removing $C_1$ makes each cycle stationary.

Now we compute $R(0,F)$. Since $R(i,F)$ is stationary across cycles, it can be written as

$$\begin{aligned}
\rho R(i,F) &= -M_s\lambda^{\overline{\Delta}^*} - a_I(\overline{\Delta}^*)\mu^2(i)/N + (1-p)\mu(i)[R(i,S)-R(i,F)] \\
&\quad +\iota[R(i+1,F)-R(i,F)], \text{ for } i \le \overline{\Delta}^*-2, \\
\rho R(\overline{\Delta}^*-1,F) &= -M_s\lambda^{\overline{\Delta}^*} - a_I(\overline{\Delta}^*)\mu^2(\overline{\Delta}^*-1)/N + \\
&\quad (1-p)\mu(\overline{\Delta}^*-1)[R(\overline{\Delta}^*-1,S)-R(\overline{\Delta}^*-1,F)] + \iota[R(0,F)-R(\overline{\Delta}^*-1,F)].
\end{aligned}$$

In the above expressions, the first two terms are the instantaneous payoff, while the next two terms are the changes in the values. Similarly, $R(i,S)$ can be expressed as

$$\begin{aligned}
\rho R(i,S) &= -M_s + \iota[R(i+1,S)-R(i,S)], \text{ for } i \le \overline{\Delta}^*-2, \\
\rho R(\overline{\Delta}^*-1,S) &= -M_s + \iota[R(0,F)-R(\overline{\Delta}^*-1,S)].
\end{aligned}$$

Solving the value functions recursively, we get

$$\begin{aligned}
R(0,F) &= \frac{1}{1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}} \times \\
&\quad \{\sum_{j=0}^{\overline{\Delta}^*-1}[\frac{-M_s\lambda^{\overline{\Delta}^*} - a_I(\overline{\Delta}^*)\mu^2(j)/N - (1-p)\mu(j)\frac{M_s}{\rho}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*-j}]}{\rho+(1-p)\mu(j)+\iota}\prod_{z=0}^{j-1}\frac{\iota}{\rho+(1-p)\mu(z)+\iota}]\}
\end{aligned} \tag{8}$$

Combining equations (7) and (8), we get the expression of $W(p)$:

$$\begin{aligned}
W(p) &= (\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}[C_1 + R(0,F)] \\
&= \frac{M_s}{\rho}\frac{(\frac{\lambda\iota}{\rho+\iota})^{\overline{\Delta}^*}}{1-(\frac{\lambda\iota}{\rho+\iota})^{\overline{\Delta}^*}}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}] - \frac{(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}}{1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}} \\
&\quad \times\{\sum_{j=0}^{\overline{\Delta}^*-1}[\frac{M_s\lambda^{\overline{\Delta}^*} + a_I(\overline{\Delta}^*)\mu^2(j)/N + (1-p)\mu(j)\frac{M_s}{\rho}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*-j}]}{\rho+(1-p)\mu(j)+\iota}\prod_{z=0}^{j-1}\frac{\iota}{\rho+(1-p)\mu(z)+\iota}]\}
\end{aligned} \tag{9}$$

### 4.2 Two major effects

Recall that, by previous results, the number of possible equilibrium cycle length $\overline{\Delta}^*$ is finite. Given that $p$ is continuous, for each possible $\overline{\Delta}^*$ there is a range of $p$ such that all $p$ in this range induce the same equilibrium cycle length $\overline{\Delta}^*$. Formally, let $P(\overline{\Delta}^*) \equiv \{p$: the equilibrium cycle length is $\overline{\Delta}^*$ under $p\}$. Since $\overline{\Delta}^*$is weakly decreasing in $p$, $P(\overline{\Delta}^*)$ is an interval : $P(\overline{\Delta}^*) = [\underline{p}(\overline{\Delta}^*), \overline{p}(\overline{\Delta}^*)]$, where $\underline{p}(\overline{\Delta}^*)$ is the lower bound and $\overline{p}(\overline{\Delta}^*)$ is the upper bound. To abuse terminology, we call the equilibrium cycle length induced by the optimal IPR strength $p^o$ as the optimal cycle length, and we label it as $\overline{\Delta}^o$.

**Proposition 3** *Among all IPR strengths that induce the same equilibrium cycle length $\overline{\Delta}^*$, $p \in P(\overline{\Delta}^*)$, the discounted social welfare is the highest under the smallest IPR strength $\underline{p}(\overline{\Delta}^*)$.*

Proposition 3 implies that among all IPR strengths inducing the same equilibrium cycle length, the Southern government always tries to choose the lowest IPR strength. Define $\underline{P} \equiv \{\underline{p}$: $\underline{p} = \underline{p}(\overline{\Delta}^*)$ for some $\overline{\Delta}^* \in \Gamma\}$. Then, by Proposition 3, $p^o \in \underline{P}$, and it must be the case that $V_F(0, p^o, \overline{\Delta}^*) = F$; that is, every generation of FDI earns zero expected profit.[19] This means that generically, $p^o < 1$, or the optimal IPR protection of the South is never perfect.

The underlying reason for Proposition 3 is that, compared to social optimum, at each instant of any state $i$ Southern firms underinvest in imitation. This is because the (flow) social gain of imitation at each instant is $M_S(\lambda^{\overline{\Delta}^*} - 1)$, while the flow profit for successful imitation is only $M_S(\xi - 1)$. Moreover, the expected time length to enjoy social gain and to enjoy private profit of a successful imitation is the same. As a result, the Southern government wants to reduce $p$ as much as possible in order to induce more intensive imitation. We call this effect the "imitation" effect.

It is surprising that this result holds regardless of the number of Southern firms, $N$. Recall that as $N$ increases, the aggregate imitation intensity increases. One would think that as $N$ becomes large enough, the aggregate equilibrium imitation intensity might be bigger than the socially optimal one. However, this will never happen, and the reason is as follows. Recall that as $N$ goes to infinity, the individual equilibrium imitation intensity converges to $\frac{(1-p)V_I}{2a_I}$. Since the private prize of successful imitation, $V_I$, is less than the social gain of successful imitation, each firm still underinvests in imitation even as $N$ goes to infinity. In terms of aggregate imitation intensity, note that the total costs of imitation across different firms are additive, instead of being increasing and convex in aggregate imitation intensity. This implies that if each Southern firm underinvests in imitation, then aggregately firms underinvest in imitation as well, regardless of the number of firms $N$.

**Proposition 4** *The discounted gross utility of the Southern consumers, $(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*} C_1$, is strictly decreasing in equilibrium cycle length, $\overline{\Delta}^*$.*

The result of Proposition 4 is quite intuitive. As the equilibrium cycle length increases, consumers will get free upgrades less often, which decreases the discounted value of consumers'

[19] This implies that the possibility of making FDI in the South does not affect Northern firms' incentive to innovate.

gross utility (free upgrades). To illustrate this point more clearly, consider two equilibrium cycle lengths $\overline{\Delta}^*$ and $\overline{\Delta}^{*\prime}$, with $\overline{\Delta}^{*\prime} > \overline{\Delta}^*$. Now consider a grand cycle with length $\overline{\Delta}^*\overline{\Delta}^{*\prime}$. Note that the comparison of the two equilibrium paths are the same across different grand cycles. Now inspect a grand cycle. The first observation is that the overall upgrades within the grand cycle are the same under two different cycle lengths.[20] However, with equilibrium length $\overline{\Delta}^*$ consumers will get more frequent free upgrades (a total number of $\overline{\Delta}^{*\prime}$) of smaller steps (each upgrade has $\overline{\Delta}^*$ steps), while with equilibrium length $\overline{\Delta}^{*\prime}$ consumers will get less frequent free upgrades (a total number of $\overline{\Delta}^*$) of bigger steps (each upgrade has $\overline{\Delta}^{*\prime}$ steps). Because of discounting, more frequent upgrades lead to a higher discounted value.

Given that the equilibrium cycle length is weakly decreasing in IPR strength $p$, Proposition 4 reveals the downside of a weak IPR strength: it will increase the equilibrium cycle length and consumers will get free upgrades less frequently. Because of this "free upgrade" effect, the Southern government has an incentive to implement a higher IPR strength.

To summarize, the imitation effect identified in Proposition 3 represents the short run benefit of lowering IPR: it speeds up imitation within each cycle. On the other hand, the free upgrade effect identified in Proposition 4 represents the long run cost of lowering IPR: it increases the equilibrium cycle length and Southern consumers get less frequent free upgrades. The optimal IPR strength $p^o$ tries to balance these two opposite effects. Of course, the optimal IPR strength also depends on effects other than the two we just mentioned. For instance, an increase in equilibrium cycle length also leads to a higher price charged by FDI, which tends to reduce the social welfare of the South. This "price" effect means that the Southern government tends to choose a high IPR strength. Moreover, an increase in equilibrium cycle length might increase the cost of imitation $a_I(\Delta)$, which again tends to reduce the social welfare of the South. This "imitation cost" effect implies that the Southern government tends to choose a high IPR strength as well.

**Example 1** *$\lambda = 1.105$, $\rho = 0.05$, $\iota = 0.04$, $\xi = 1.1$, $M_S = 10$, $F = 15$, $a_I = 3$, $n = 0$, $N = 5$.*[21] *The minimum cycle length is* 2*, and the maximum cycle length is* 32*. The optimal cycle length is* 3 *and the optimal IPR strength is* 0.9089*.*

**Example 2** *$\lambda = 1.21$, $\rho = 0.05$, $\iota = 0.04$, $\xi = 1.2$, $M_S = 10$, $F = 1$, $a_I = 3$, $n = 0$, $N = 2$. The minimum cycle length is* 1*, and the maximum cycle length is* 4*. The optimal cycle length is* 2 *and the optimal IPR strength is* 0.3994*.*

The above examples share a common feature: the optimal cycle length is very close to the minimum cycle length.[22] Actually, in both examples the optimal cycle length is just one step longer than the minimum cycle length. In all the numerical simulations we run, the optimal cycle length either coincides with the minimum cycle length or is one step longer than

[20] The overall upgrades within a grand cycle is $\overline{\Delta}^*\overline{\Delta}^{*\prime}$.

[21] In the numerical examples, we assume that the cost of imitation has the following form: $a_I(\overline{\Delta})^n$. Thus $n = 0$ means that imitation cost is independent of cycle length, and $n = 1$ implies that imitation cost is linear in cycle length.

[22] Note that Condition 2 does not hold for these examples.

the minimum cycle length. This indicates that, quantitatively, the free upgrade effect is the dominant effect.

Does the North always benefit from an increase in the IPR of the South? The answer is no. To see this, consider $\underline{p}(\overline{\Delta}^*_{\min}) > \underline{p}(\overline{\Delta}^*_{\max})$. By previous results, at both $\underline{p}(\overline{\Delta}^*_{\min})$ and $\underline{p}(\overline{\Delta}^*_{\max})$, $V_F(0, \underline{p}(\overline{\Delta}^*_{\min}), \overline{\Delta}^*_{\min}) = F = V_F(0, \underline{p}(\overline{\Delta}^*_{\max}), \overline{\Delta}^*_{\max})$. Thus, Northern firms always get an expected zero payoff at both $\underline{p}(\overline{\Delta}^*_{\min})$ and $\underline{p}(\overline{\Delta}^*_{\max})$. Therefore, the North is indifferent between these two Southern IPRs. It is not difficult to construct an example in which the North actually is worse off when the Southern IPR increases. For instance, consider $\underline{p}(\overline{\Delta}^*_{\min}) - \varepsilon < \underline{p}(\overline{\Delta}^*_{\min})$, where $\varepsilon > 0$ is small. With $p = \underline{p}(\overline{\Delta}^*_{\min})$, the equilibrium cycle length is $\overline{\Delta}^*_{\min}$ and Northern firms always get an expected zero payoff. With $p = \underline{p}(\overline{\Delta}^*_{\min}) - \varepsilon$, the equilibrium cycle length is $\overline{\Delta}^*_{\min} + 1$ and Northern firms get a positive expected payoff when they make FDI. Therefore, the North is better off with a lower Southern IPR. However, fixing the equilibrium cycle length (among the $p$s that induce the same equilibrium cycle length), Northern firms always benefit from a strengthening of the Southern IPR. This is because, by (6), $V_F(0, p, \overline{\Delta}^*)$ is increasing in $p$. The following proposition summarizes the above results.

**Proposition 5** *(i) Fixing the equilibrium cycle length $\overline{\Delta}^*$, the North always benefits from an increase in $p$. (ii) If an increase in the Southern IPR $p$ causes a decrease in the equilibrium cycle length $\overline{\Delta}^*$, the North could be worse off.*

It is surprising that the North does not always benefit from an increase in the Southern IPR. If FDI occurs only once (or the cycle length is fixed), then the North always benefits from a strengthening in the Southern IPR, since it prolongs the monopoly of the (existing) FDI in the South. However, if an increase in the Southern IPR reduces the equilibrium cycle length, FDI would occur more frequently, which reduces the expected length of the monopoly of any existing FDI. This effect tends to reduce the profit of Northern FDI and make the North worse off. Intuitively, by reducing the equilibrium cycle length, an increase in the Southern IPR intensifies the competition among different generations of the Northern FDIs, which could potentially make the North worse off but benefits the Southern consumers.

### 4.3 Comparative statics

Now we investigate how changes in various parameter values affect the optimal IPR strength. However, general analytical results are difficult to derive for the following two reasons. First, the expression of $W(p)$ (equation (9)) is quite complicated. Second, the equilibrium cycle length is discrete. Due to these difficulties, we proceed in two steps. First, we derive analytical results for the case that the optimal cycle length does not change as parameter values change. This case is not knife-edge as the equilibrium cycle length is discrete, while all the parameter values are continuous. Second, we use simulations to demonstrate how the optimal cycle length and the optimal IPR change as parameter values change.

**Proposition 6** *Suppose the optimal cycle length $\overline{\Delta}^o$ does not change. (i) When the step size of the quality ladder, $\lambda$, increases, the optimal IPR strength $p^o$ decreases, and the equilibrium imitation intensity increases. (ii) When the marginal cost of FDI production, $\xi$, increases, the* *optimal IPR strength $p^o$ increases. (iii) When the size of the Southern market, $M_S$, increases, the optimal IPR strength $p^o$ decreases, and the equilibrium imitation intensity increases. (iv) When the number of Southern firms, $N$, increases, the optimal IPR strength $p^o$ increases.*

The results of Proposition 6 are straightforward given the comparative statics results in Proposition 2. For example, as the size of the Southern market increases, any FDI becomes more profitable. Given that the optimal cycle length does not change, the Southern government now can reduce its IPR strength. A lower IPR strength and a bigger market imply a higher aggregate imitation intensity. This result implies that countries with bigger markets, such as China, tend to have lower IPR strength and higher aggregate imitation intensity. Part (iv) of Proposition 6 implies that countries with a more competitive industry tend to have a higher IPR strength.

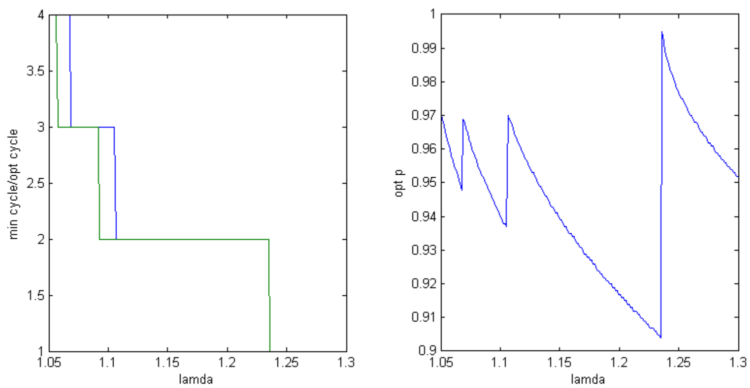


Figure 1: Optimal Cycle Length and IPR as $\lambda$ Changes-1

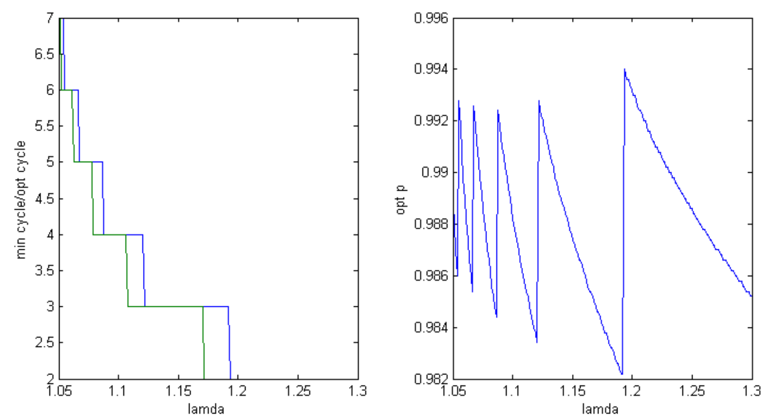


Figure 2: Optimal Cycle Length and IPR as $\lambda$ Changes-2

Figure 1 and Figure 2 illustrate how the optimal cycle length (the left panels) and the optimal IPR strength (the right panels) change as $\lambda$, the step size of the quality ladder, varies. The parameter values for Figure 1 are: $\rho = 0.05$, $\iota = 0.04$, $\xi = 1.1$, $M_S = 10$, $F = 15$, $a_I = 3$, $n = 0$, and $N = 10$. And those for Figure 2 are: $\rho = 0.05$, $\iota = 0.02$, $\xi = 1.1$, $M_S = 10$, $F = 50$, $a_I = 0.3$, $n = 1$, $N = 20$.[23] The two figures exhibit the same pattern. As $\lambda$ increases, both the minimum cycle length and the optimal cycle length weakly decrease. Moreover, the optimal cycle length either coincides or is one step bigger than the minimum cycle length. As to the optimal IPR strength, it monotonically decreases as $\lambda$ increases when the optimal cycle length remains the same; and it jumps up discretely when an increase in $\lambda$ causes a decrease in the optimal cycle length.

Recall that an increase in $\lambda$ directly increases the price charged by the FDI. Thus the minimum cycle length is weakly decreasing in $\lambda$. Another pattern worth mentioning is that, as $\lambda$ increases, when the minimum cycle length decreases the optimal cycle length does not decrease immediately. It decreases to the minimum cycle length only when $\lambda$ increases further by some amount. In other words, the decreases in the optimal cycle length "lag behind" those of the minimum cycle length. To understand this pattern, note that when the minimum cycle length decreases by one, at that particular $\lambda$ for the optimal cycle length to match the minimum cycle length (decreases by one as well) the IPR strength $p$ has to be 1 (by the definition of the minimum cycle length). But $p = 1$ implies a complete shutdown of imitation. However, since the cost of imitation is convex in imitation intensity, the cost of imitation approaches 0 faster than the social return. This implies that the social return of imitation is very big relative to the cost of imitation when the imitation intensity goes to 0. As a result, in the neighborhood of that particular $\lambda$ the social planner would optimally choose not to increase $p$ all the way to 1, though a smaller cycle length is feasible. In other words, the imitation effect outweighs the free upgrade effect, leading to a one-step gap between the optimal and the minimum cycle length.

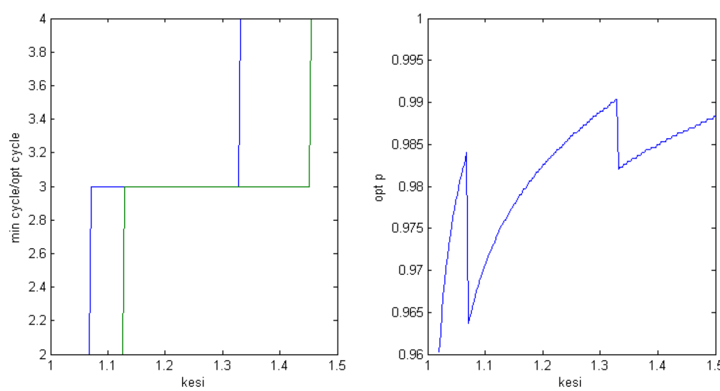


Figure 3: Optimal Cycle Length and IPR as $\xi$ Changes-1

[23] In the examples, $\lambda < \xi$ for some value of $\lambda$. But $\lambda^{\overline{\Delta}} > \xi$ always hold for all $\lambda$, which means that the price charged by FDI is always higher than that of a successful imitator.

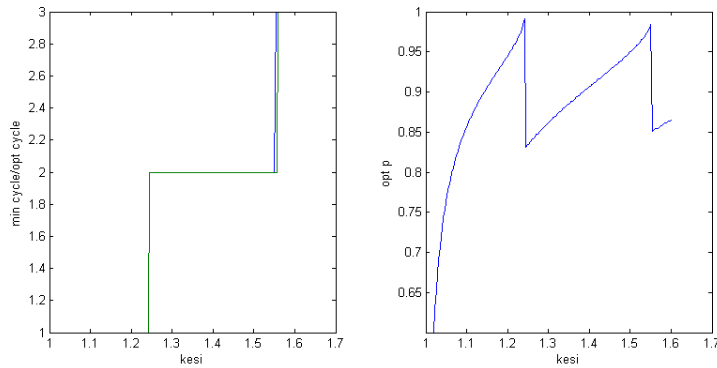


Figure 4: Optimal Cycle Length and IPR as $\xi$ Changes-2

Figure 3 and Figure 4 illustrate the impacts of changes in FDI's production cost $\xi$. The parameter values for Figure 3 are: $\lambda = 1.2$, $\rho = 0.05$, $\iota = 0.04$, $M_S = 10$, $F = 50$, $a_I = 1$, $n = 0$, and $N = 10$. And those for Figure 4 are: $\lambda = 1.25$, $\rho = 0.05$, $\iota = 0.02$, $M_S = 10$, $F = 1$, $a_I = 2$, $n = 1$, $N = 20$. The two figures exhibit the same pattern. As $\xi$ increases, both the minimum cycle length and the optimal cycle length weakly increase, and they are at most one step apart. As to the optimal IPR strength, it monotonically increases as $\xi$ increases when the optimal cycle length remains the same; and it jumps down discretely when an increase in $\xi$ causes an increase in the optimal cycle length.

It is easy to understand why an increase in $\xi$ leads the minimum cycle length to increase. This is because, as pointed out earlier, an increase in $\xi$ directly reduces the profitability of FDI. Another pattern worth mentioning is that, as $\xi$ increases, the increases in the optimal cycle length "precede" those of the minimum cycle length. Again, the underlying reason is that the social return of imitation is very big relative to the cost of imitation when the imitation intensity is close enough to 0. When $\xi$ is close enough to the next "jump" point of $\xi$ at which the minimum cycle length jumps up, to match the equilibrium cycle length to the minimum cycle length the IPR strength has to be very close to 1. Since the social return of imitation is relatively very high when $\mu$ is close to 0, the optimal cycle length jumps up "earlier" than the minimum cycle length does to ensure that the imitation intensity is not close to 0.

Figure 5 and Figure 6 illustrate the impacts of changes in $\iota$, the Northern innovation rate. The parameter values for Figure 5 are: $\lambda = 1.2$, $\rho = 0.05$, $\xi = 1.1$, $M_S = 10$, $F = 50$, $a_I = 3$, $n = 0$, and $N = 20$. And those for Figure 6 are: $\lambda = 1.2$, $\rho = 0.05$, $\xi = 1.1$, $M_S = 10$, $F = 8$, $a_I = 2$, $n = 1$, $N = 10$. As $\iota$ increases, both the minimum cycle length and the optimal cycle length weakly increase, and they are at most one step apart. Moreover, the optimal cycle length either coincides or is one step bigger than the minimum cycle length (in the second figure, they always coincide). As to the optimal IPR strength, when the optimal cycle length remains the same it is increasing in $\iota$ when $\iota$ is low and it is decreasing in $\iota$ when $\iota$ is high; and it jumps down discretely when an increase in $\iota$ causes an increase in the optimal cycle length.

As mentioned earlier, an increase in $\iota$ directly reduces the expected length of FDI monopoly, thus reducing the profitability of FDI. This is the reason why the minimum cycle length is increasing in $\iota$. To understand the relationship between the optimal $p^o$ and $\iota$ when the

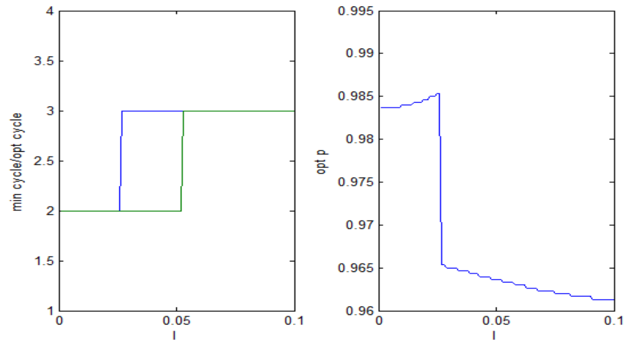


Figure 5: Optimal Cycle Length and IPR as $\iota$ Changes-1

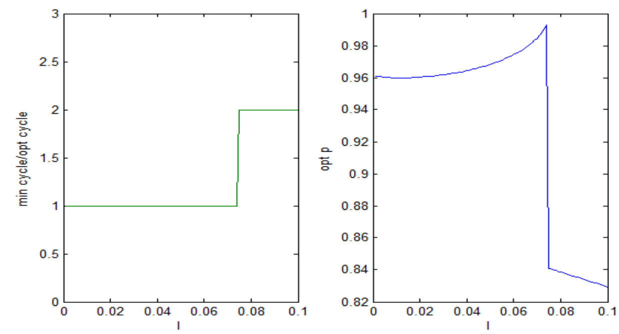


Figure 6: Optimal Cycle Length and IPR as $\iota$ Changes-2

optimal cycle length remains the same, note that an increase in $\iota$ also dampens the incentive of imitation, which indirectly makes FDI more profitable. How big this indirect effect is depends on the intensity of imitation. When $p$ is lower, the imitation intensity is higher, and this effect tends to be stronger. Because the optimal $p^o$ is lower when $\iota$ is higher, this indirect effect outweighs the direct effect when $\iota$ is high and the opposite happens when $\iota$ is low.

Figures 7 and 8 illustrate the impacts of changes in the market size $M_S$. The parameter values for Figure 7 are: $\lambda = 1.2$, $\rho = 0.05$, $\iota = 0.02$, $\xi = 1.1$, $F = 50$, $a_I = 3$, $n = 1$, and $N = 20$. And those for Figure 8 are: $\lambda = 1.2$, $\rho = 0.05$, $\iota = 0.03$, $\xi = 1.1$, $F = 12$, $a_I = 0.3$, $n = 0$, $N = 10$. As $M_S$ increases, both the minimum cycle length and the optimal cycle length weakly decrease, and they are at most one step apart. As to the optimal IPR strength, it monotonically decreases as $M_S$ increases when the optimal cycle length remains the same; and it jumps up discretely when an increase in $M_S$ causes a decrease in the optimal cycle length. The decreases in the optimal cycle length "lag behind" the decreases in the minimum cycle length, for the same reason as mentioned before.

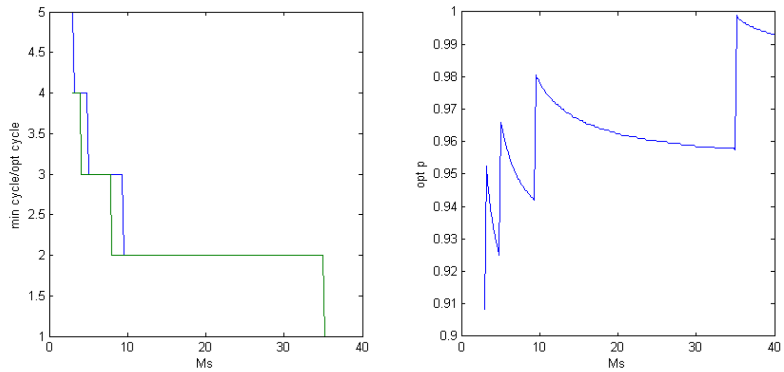


Figure 7: Optimal Cycle Length and IPR as $M_S$ Changes-1

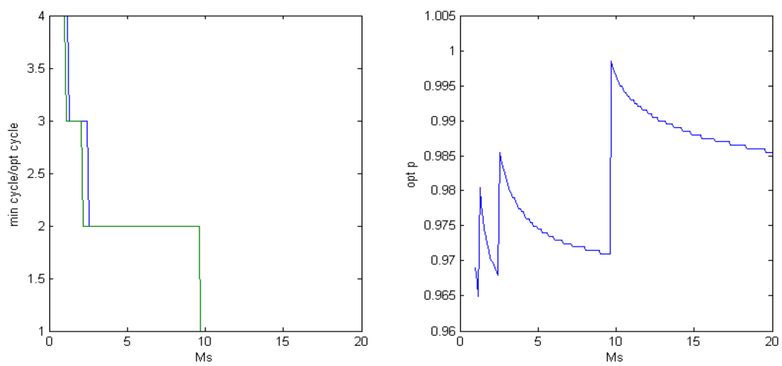


Figure 8: Optimal Cycle Length and IPR as $M_S$ Changes-2

The above pattern generates two testable empirical implications. First, a developing country with a bigger market size tends to have a smaller FDI cycle length. Second, among developing countries the relationship between the domestic market size and the optimal IPR strength is non-monotonic. For developing countries having the same FDI cycle length, the IPR strength is decreasing in the market size. But a bigger market size implies a smaller FDI cycle length, which tends to increase the IPR strength.

Figure 9 shows the relationship between IPR and the log of GDP between 1995-2005 for 79 developing countries.[24] In the figure, the loess-fit curve clearly indicates that the relationship between IPR and market size is not monotonic. Actually, the shape of the loess-fit curve largely resembles that in the right panels of Figures 7 and 8. Therefore, this shows that the prediction of our model is largely consistent with empirical evidence.[25]

[24] The IPR data is obtained from Park (2008). The GDP data is based on Penn World Tables: https://pwt.sas.upenn.edu/php_site/pwt71/pwt71_form.php

[25] Empirically, Auriol et al. (2012) found that IPR and domestic market size among developing countries have a U-shaped relationship.

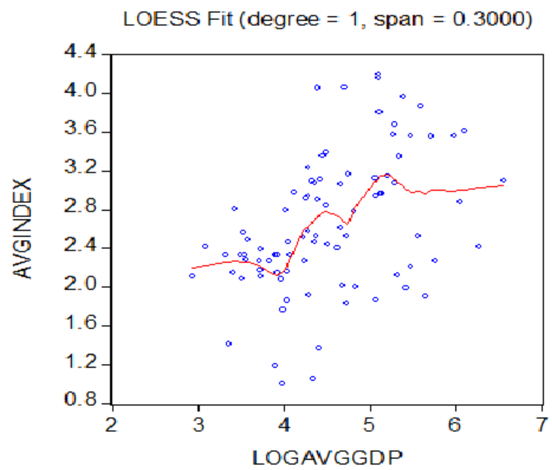


Figure 9: The Relationship between IPR and GDP

Figure 10 illustrates the impacts of changes in the imitation cost $a_I$. The parameter values are $\lambda = 1.26$, $\rho = 0.05$, $\iota = 0.02$, $\xi = 1.25$, $M_S = 10$, $F = 10$, $n = 1$ and $N = 5$. The optimal cycle length does not change as $a_I$ varies, and the optimal IPR strength is monotonically decreasing in $a_I$. The reason that the optimal cycle length does not change with $a_I$ is that the minimum cycle length is independent of $a_I$.

To investigate the impacts of the relationship between imitation cost and the North-South technology gap, we consider the following example: $\lambda = 1.2$, $\rho = 0.05$, $\iota = 0.02$, $\xi = 1.1$, $M_S = 10$, $F = 50$. In the first scenario, the imitation cost is independent of the cycle length: $a_I = 6$. In the second scenario, the imitation cost is linear in cycle length $a_I = 3\overline{\Delta}$. Note that the imitation cost is the same when the cycle length is 2, and the imitation cost is higher in the second scenario when $\overline{\Delta} > 2$ and the opposite is true when $\overline{\Delta} = 1$. The three dimensional figures (varying $M_S$ and $N$) are illustrated in Figures 11 and 12.

As we can see from the figures, the patterns of the optimal cycle length (the left panels) are almost the same under two different cost structures. This is again due to the following two facts. First, the dominance of the free upgrade effect implies that the optimal cycle length either coincides with or is very close to the minimum cycle length. Second, the minimum cycle length is independent of the imitation cost. From the right panels of Figures 11 and 12, we can see that the optimal IPR is lower under the second cost structure when the optimal cycle length is bigger than 2, and it is higher under the second cost structure when the optimal cycle length is 1. This is because the optimal cycle length is almost the same under two cost structures, thus the optimal IPR depends only on the magnitude of the imitation costs.

To summarize, lower imitation costs will lead to higher optimal IPR strength. Since imitation costs are lower in more technologically advanced economies, this implies that IPR strength should be increasing in GDP per capita among developing countries. Empirically, Maskus (2000), Braga et al. (2000), and Chen and Puttitanun (2005) all found a U-shaped re-

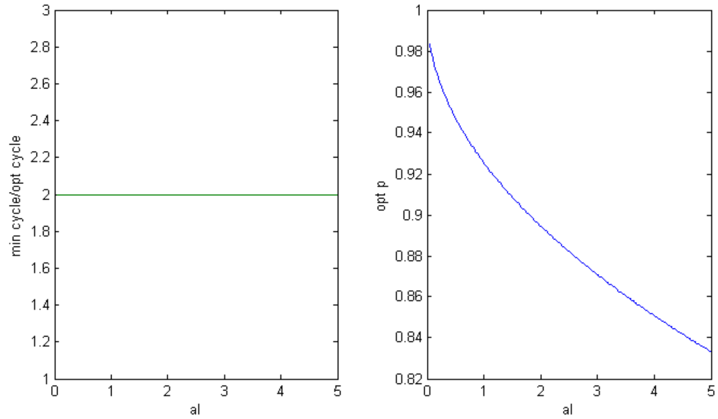


Figure 10: Optimal Cycle Length and IPR as $a_I$ Changes

lationship between IPR and per capita income. However, the negative relationship only holds for countries with very low income levels. For the majority of developing countries, IPR is increasing in income per capita.[26]

In Figure 13, we plot the relationship between IPR and the log of GDP per capita between 1995-2005 for 79 developing countries.[27] In the figure, the loess-fit curve clearly indicates that IPR and GDP per capita are positively correlated. Compared to Figure 9, the loess-fit curve in Figure 13 is much steeper. This implies that IPR increases with the level of economic development, while the relationship between IPR and domestic market size is less clear-cut.[28] This pattern is largely consistent with the prediction of our model.

Finally, Figures 14 and 15 illustrate the impacts of changes in $N$, the number of Southern firms. The parameter values for Figure 9 are: $\lambda = 1.105$, $\rho = 0.05$, $\iota = 0.04$, $\xi = 1.1$, $M_S = 10$, $F = 15$, $a_I = 3$, and $n = 0$. And those for Figure 10 are: $\lambda = 1.26$, $\rho = 0.05$, $\iota = 0.02$, $\xi = 1.25$, $M_S = 10$, $F = 10$, $a_I = 2$, and $n = 1$. In both figures, the optimal cycle length does not change as $N$ varies: while in the second figure the optimal cycle length always coincides with the minimum cycle length, in the first figure the optimal cycle length is always one step above the minimum cycle length. As to the optimal IPR strength, it monotonically increases as $N$ increases.

To understand why the optimal cycle length does not change with $N$, note that the minimum cycle length is independent of $N$ (when $p = 1$, $N$ does not affect the profitability of the FDI). Given that the optimal cycle length "closely follows" the minimum cycle length, it implies that the optimal cycle length does not change with $N$. One implication for this pattern is that, other things equal, Southern countries with more competitive industries will set a higher IPR strength.

[26] In Chen and Puttitanun (2005), IPR reaches its minimum for countries with a GDP per capita of \$854 in 1995 prices.

[27] The source is the same as that of Figure 9.

[28] A big domestic market can be either due to a large population, or due to a high GDP per capita.

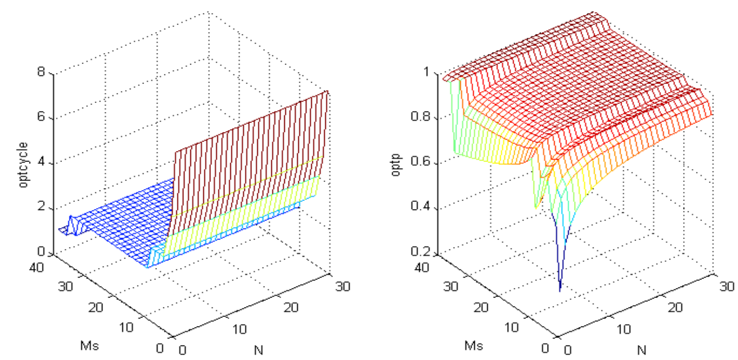


Figure 11: Optimal Cycle Length and Optimal IPR with Cost Structure 1

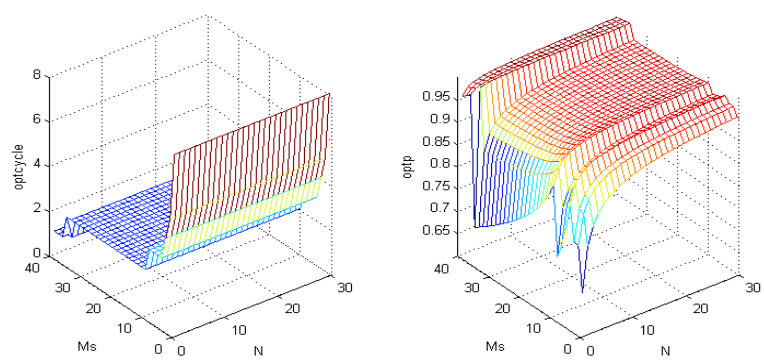


Figure 12: Optimal Cycle Length and Optimal IPR with Cost Structure 2

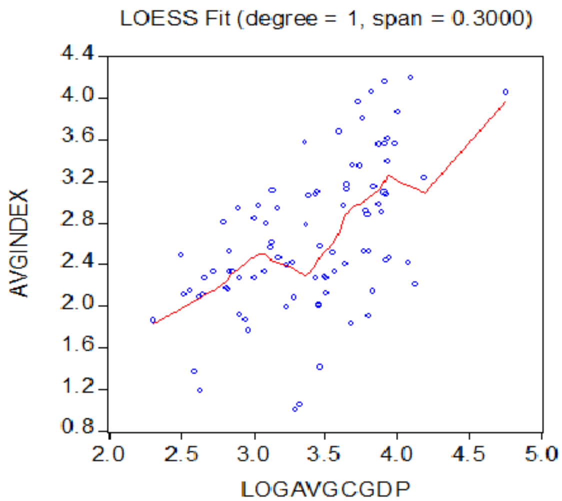


Figure 13: The Relationship between IPR and GDP Per Capita

### 4.4 Alternative objectives

Now we modify the objective of the Southern government. Suppose its objective is to maximize the weighted sum of consumer surplus and producer surplus. More specifically, $w_t = \alpha CS_t + PS_t$, where $\alpha \in (0, 1]$ is the relative weight between consumer surplus and producer surplus. We assume $\alpha < 1$ because usually the Southern countries care relatively more about firms and less about consumers. Correspondingly, the discounted (weighted) social welfare becomes

$$\begin{aligned} W(p) &= (\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}[C_1 + R(0, F)] \\ &= \frac{\alpha M_s}{\rho}\frac{(\frac{\lambda\iota}{\rho+\iota})^{\overline{\Delta}^*}}{1-(\frac{\lambda\iota}{\rho+\iota})^{\overline{\Delta}^*}}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}] - \frac{(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}}{1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}} \\ &\quad \times\{\sum_{j=0}^{\overline{\Delta}^*-1}[\frac{\alpha M_s \lambda^{\overline{\Delta}^*} + a_I(\overline{\Delta}^*)\mu^2(j)/N + (1-p)\mu(j)\frac{M_s[1-(1-\alpha)\xi]}{\rho}[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*-j}]}{\rho+(1-p)\mu(j)+\iota} \\ &\quad \times\prod_{z=0}^{j-1}\frac{\iota}{\rho+(1-p)\mu(z)+\iota}]\}. \end{aligned} \tag{10}$$

Figures 16 and 17 illustrate the impacts of changes in $\alpha$. The parameter values of the figures are the same as those when $N$ changes (Figures 9 and 10, with $N = 10$). In both figures, the optimal cycle length does not change with $\alpha$. Given that the optimal cycle length is invariant to $\alpha$, it follows that the optimal $p^o$ does not change with $\alpha$ either, as the weight $\alpha$ does not affect the profitability of FDI. To understand why the optimal cycle length is invariant to $\alpha$, first note

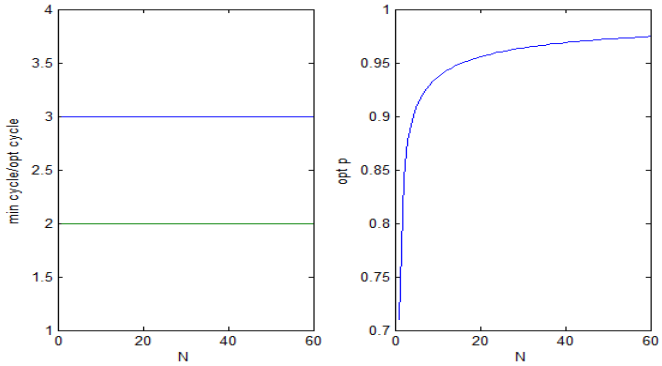


Figure 14: Optimal Cycle Length and IPR as $N$ Changes-1

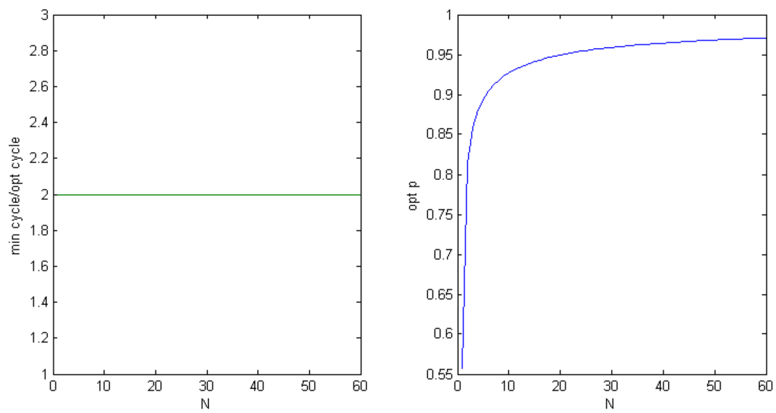


Figure 15: Optimal Cycle Length and IPR as $N$ Changes-2

that the minimum cycle length is independent of $\alpha$ since $\alpha$ has no impact on the profitability of FDI. As $\alpha$ decreases, the Southern government cares less about consumer welfare, hence the free upgrade effect becomes less important. Thus one would expect the optimal cycle length becomes further away from the minimum cycle length. However, the imitation effect becomes less important as well as $\alpha$ decreases. To see this, observe the second term (in the bracket) of (10). The social return of imitation is roughly measured by the difference between $\alpha M_s \lambda^{\overline{\Delta}^*}$ (the price that consumers pay before imitation) and $M_s[1-(1-\alpha)\xi]$ (the flow social welfare after successful imitation), and this difference is obviously increasing in $\alpha$. It turns out that the magnitude of the free upgrade effect and that of the imitation effect decrease with $\alpha$ roughly at the same rate, leading to the optimal cycle length invariant to $\alpha$. This serves as a robustness check, which implies that all the previous comparative statics results (with $\alpha = 1$) will also hold when the relative weight on consumer surplus changes.

Alternatively, the South government's objective could be to minimize the expected $G_{NS}$,

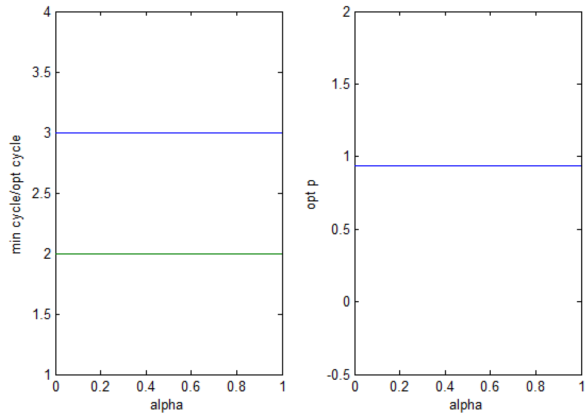


Figure 16: Optimal Cycle Length and IPR as $\alpha$ Changes-2

the technology gap between the leading Northern firm and the leading Southern firm. Note that $G_{NS}$ is a random variable and it is isomorphic across cycles.[29] In this alternative setting, there are two effects parallel to the free upgrade effect and the imitation effect. Specifically, an increase in the cycle length tends to increase the technology gap $G_{NS}$.[30] Thus, this cycle length effect implies that the South should implement a strong IPR in order to reduce the cycle length. On the other hand, an increase in imitation intensity tends to decrease $G_{NS}$.[31] Thus, this imitation effect entails a weak IPR. The optimal IPR should balance these two opposite effects. Based on the above analysis, we expect the optimal IPR in this alternative setting exhibits similar qualitative features as those in the benchmark setting when the Southern government objective is to maximize the discounted social welfare.

# 5 Conclusion

This paper develops a quality ladder model in which the technology gap between the North and the South is endogenously determined. Equilibrium exhibits FDI cycles: New FDI arrives if and only if the technology gap reaches some threshold. A stronger IPR in the South discourages imitation and reduces the FDI cycle length. A smaller market size and more imitating firms in the South tend to enlarge the equilibrium FDI cycle length. A weaker IPR in the South brings a short-run benefit: within each FDI cycle it encourages imitation and increases the Southern welfare. However, it entails a long run cost: it increases the FDI cycle length and

[29] At each instant, $G_{NS}$ can be decomposed into two components: the technology gap between the leading Southern firm and the most recent FDI, and the technology gap between the most recent FDI and the leading Northern firm.

[30] This increases the technology gap between the most recent FDI and the leading Southern firm before successful imitation.

[31] Successful imitation reduces the technology gap between the most recent FDI and the leading Southern firm to zero.

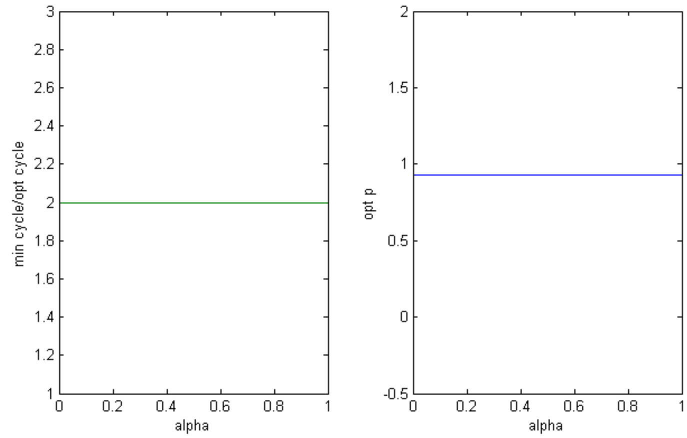


Figure 17: Optimal Cycle Length and IPR as $\alpha$ Changes-2

makes FDI less frequent, which due to discounting reduces the Southern welfare. The optimal IPR strength balances these two effects. Our comparative statics results show that the optimal IPR is non-monotonic in the domestic market size, and is increasing in the level of economic development. These two predictions are largely consistent with empirical evidence. Moreover, IPR is positively correlated with the number of firms in the Southern industry. Finally, we found that the North does not always benefit from an increase in the Southern IPR, which is a surprising result.

In our model there is a single industry. It is straightforward to extend our model to multiple industries. Given that the situations of different industries are likely to be different (have different parameter values), the optimal IPR in the South could be industry-specific. Indeed, the IPR in different industries in the same developing country can be different, and it can be achieved by varying the law enforcement across industries. Another possible extension is to consider multiple developing countries who are competing with each other in attracting FDI. With a limited supply of FDI, each country's IPR policy imposes externalities on other countries. We leave this for future research.

# Appendix

**Proof of Lemma 1.**

**Proof.** Part (i). Define

$$H(\mu_I) \equiv a_I(2N-1)\mu_I^2 + [2a_I\frac{\rho}{1-p} - (N-1)(1-p)V_I]\mu_I - V_I\rho. \tag{11}$$

By equation (3), $H(\mu_I^*) = 0$. Note that $H(\mu_I)$ is a quadratic function in $\mu_I$, the coefficient of $\mu_I^2$ is strictly bigger than 0, and $H(0) < 0$. This implies that $H(\mu_I) = 0$ has exactly one positive solution, or there is a unique $\mu_I^* > 0$. To show that $\mu_I^* < \frac{(1-p)V_I}{2a_I}$, we compute $H(\frac{(1-p)V_I}{2a_I})$:

$$H(\frac{(1-p)V_I}{2a_I}) = \frac{(1-p)^2V_I^2}{4a_I} > 0.$$

Combining with the fact that $H(0) < 0$, we must have $\mu_I^* < \frac{(1-p)V_I}{2a_I}$. On the other hand,

$$H(\frac{(N-1)(1-p)V_I}{(2N-1)a_I}) = -\frac{V_I\rho}{2N-1} < 0.$$

Combining with the fact that $H(0) < 0$, we must have $\mu_I^* > \frac{(N-1)(1-p)V_I}{(2N-1)a_I}$.

Part (ii). We show the properties hold for $\mu_I^*$. It immediately follows that the properties also hold for $\mu^*$, as $\mu^* = N\mu_I^*$.

Suppose $V_I' > V_I$, and define $H(\mu_I^*, V_I) = 0$ and $H(\mu_I^{*\prime}, V_I') = 0$. Inspecting (11), we can see that, for all $\mu_I \geq 0$, $H(\mu_I, V_I') < H(\mu_I, V_I)$. This implies that $H(\mu_I^*, V_I') < 0$. Therefore, $\mu_I^{*\prime} > \mu_I^*$. This proves that $\mu_I^*$ is increasing in $V_I$.

Suppose $p' > p$, and define $H(\mu_I^*, p) = 0$ and $H(\mu_I^{*\prime}, p') = 0$. Inspecting (11), we can see that, for all $\mu_I \geq 0$, $H(\mu_I, p') > H(\mu_I, p)$. This implies that $H(\mu_I^{*\prime}, p) < 0$. Therefore, $\mu_I^{*\prime} < \mu_I^*$. This proves that $\mu_I^*$ is decreasing in $p$.

Suppose $a_I' > a_I$, and define $H(\mu_I^*, a_I) = 0$ and $H(\mu_I^{*\prime}, a_I') = 0$. Inspecting (11), we can see that, for all $\mu_I \geq 0$, $H(\mu_I, a_I') > H(\mu_I, a_I)$. This implies that $H(\mu_I^{*\prime}, a_I) < 0$. Therefore, $\mu_I^{*\prime} < \mu_I^*$. This proves that $\mu_I^*$ is decreasing in $a_I$.

Suppose $N' = N + 1$, and define $H(\mu_I^*, N) = 0$ and $H(\mu_I^{*\prime}, N') = 0$. Computing the difference, $H(\mu_I, N') - H(\mu_I, N) = [2a_I\mu_I - (1-p)V_I]\mu_I$. Thus, for all $\mu_I \in (0, \frac{(1-p)V_I}{2a_I})$, $H(\mu_I, N') - H(\mu_I, N) < 0$. Since, by part (i), $\mu_I^* \in (0, \frac{(1-p)V_I}{2a_I})$ and $\mu_I^{*\prime} \in (0, \frac{(1-p)V_I}{2a_I})$, it implies that $H(\mu_I^*, N') < 0$. Therefore, we must have $\mu_I^{*\prime} > \mu_I^*$. This proves that $\mu_I^*$ is increasing in $N$. ■

**Proof of Lemma 2.**

**Proof.** Part (i). Since $V_I(0, \overline{\Delta})$ is increasing in $\overline{\Delta}$ and $V_I(\overline{\Delta}-1, \overline{\Delta})$ is independent of $\overline{\Delta}$, by Lemma 1, it is sufficient to show that $\lim_{\overline{\Delta}\to\infty} \mu_I^*(0, \overline{\Delta}) \leq \lambda\mu_I^*(\overline{\Delta}-1, \overline{\Delta})$. The corresponding $V_I$s are $M_S(\xi-1)/\rho$ and $M_S(\xi-1)/(\rho+\iota)$, respectively. By equation (3),

$$\begin{aligned}
& \lambda\mu_I(\overline{\Delta}-1, \overline{\Delta}) - \lim_{\overline{\Delta}\to\infty} \mu_I(0, \overline{\Delta}) \\
\propto\ & (N-1)(1-p)M_S(\xi-1)(\frac{\lambda}{\rho+\iota} - \frac{1}{\rho}) \\
& +\lambda\sqrt{[2a_I\frac{\rho}{1-p} - (N-1)(1-p)M_S(\xi-1)\frac{1}{\rho+\iota}]^2 + 4M_S(\xi-1)\frac{\rho}{\rho+\iota}a_I(2N-1)} \\
& -\sqrt{[2a_I\frac{\rho}{1-p} - (N-1)(1-p)M_S(\xi-1)\frac{1}{\rho}]^2 + 4M_S(\xi-1)a_I(2N-1)}.
\end{aligned}$$

Since by Condition 2, $\lambda\rho > \rho + \iota$. The above expression being greater than 0 is equivalent to the second term minus the 3rd term being positive, which can be simplified as

$$4M_S(\xi-1)a_I(2N-1)(\frac{\lambda^2\rho}{\rho+\iota} - 1) - 4M_S(\xi-1)a_I\rho(N-1)(\frac{\lambda^2}{\rho+\iota} - \frac{1}{\rho}) > 0.$$

The above inequality holds obviously.

Part (ii). It is enough to show that $V_F(0,\overline{\Delta}+1) - V_F(0,\overline{\Delta}) > 0$ for all $\overline{\Delta}$. Note that $V_I(i+1,\overline{\Delta}+1) = V_I(i,\overline{\Delta})$ for all $i \le \overline{\Delta}-1$. And because $a_I(\overline{\Delta}+1) \ge a_I(\overline{\Delta})$, following part (ii) of Lemma 1 we have $\mu(i+1,\overline{\Delta}+1) \le \mu(i,\overline{\Delta})$. Let

$$x(i,\overline{\Delta}) \equiv \sum_{j=i}^{\overline{\Delta}-1}[\frac{1}{\rho+(1-p)\mu(j,\overline{\Delta})+\iota}\prod_{k=i}^{j-1}\frac{\iota}{\rho+(1-p)\mu(k,\overline{\Delta})+\iota}.$$

By the fact that $\mu(i+1,\overline{\Delta}+1) \le \mu(i,\overline{\Delta})$ we have $x(i+1,\overline{\Delta}+1) \ge x(i,\overline{\Delta})$. Since $\mu(i,\overline{\Delta}) \ge \mu(\overline{\Delta}-1,\overline{\Delta})$, we have $x(i,\overline{\Delta}) < \frac{1}{\rho+(1-p)\mu(\overline{\Delta}-1,\overline{\Delta})}$.

By (6), $V_F(0,\overline{\Delta}+1) - V_F(0,\overline{\Delta}) > 0$ is equivalent to

$$\frac{\lambda^{\overline{\Delta}+1}-\xi}{\lambda^{\overline{\Delta}}-\xi} > \frac{x(0,\overline{\Delta})}{x(0,\overline{\Delta}+1)}.$$

Since $\frac{\lambda^{\overline{\Delta}+1}-\xi}{\lambda^{\overline{\Delta}}-\xi} > \lambda$, it is enough to show $\lambda x(0,\overline{\Delta}+1) \ge x(0,\overline{\Delta})$, which is equivalent to

$$\frac{\lambda}{\rho+(1-p)\mu(0,\overline{\Delta}+1)+\iota} + \frac{\lambda\iota}{\rho+(1-p)\mu(0,\overline{\Delta}+1)+\iota}x(1,\overline{\Delta}+1) - x(0,\overline{\Delta}) \ge 0.$$

Since $x(i+1,\overline{\Delta}+1) \ge x(i,\overline{\Delta})$, the following inequality is sufficient:

$$\frac{\lambda}{\rho+(1-p)\mu(0,\overline{\Delta}+1)+\iota} + [\frac{\lambda\iota}{\rho+(1-p)\mu(0,\overline{\Delta}+1)+\iota} - 1]x(0,\overline{\Delta}) \ge 0.$$

Since $\lambda\iota \le \rho+\iota$, the fact that $x(i,\overline{\Delta}) < \frac{1}{\rho+(1-p)\mu(\overline{\Delta}-1,\overline{\Delta})}$ implies that the following is enough

$$\begin{aligned} &\frac{\rho+(1-p)\mu(0,\overline{\Delta}+1)+\iota-\lambda\iota}{\rho+(1-p)\mu(\overline{\Delta}-1,\overline{\Delta})} \le \lambda \\ \Leftrightarrow\quad &\rho+\iota+(1-p)\mu(0,\overline{\Delta}+1) \le \lambda(\rho+\iota)+\lambda(1-p)\mu(\overline{\Delta}-1,\overline{\Delta}). \end{aligned}$$

Now the following condition is sufficient: $\mu(0,\overline{\Delta}+1) \le \lambda\mu(\overline{\Delta}-1,\overline{\Delta})$. Since $\mu(\overline{\Delta},\overline{\Delta}+1) \le \mu(\overline{\Delta}-1,\overline{\Delta})$,

$$\mu(0,\overline{\Delta}+1) \le \lambda\mu(\overline{\Delta}-1,\overline{\Delta}) \Leftarrow \mu(0,\overline{\Delta}+1) \le \lambda\mu(\overline{\Delta},\overline{\Delta}+1).$$

But the last inequality holds by part (i). ■

**Proof of Proposition (2).**

**Proof.** Part (i). As either $p$ or $a_I$ increases, by part (ii) of Lemma 1, $\mu(i,\overline{\Delta})$ decreases for all $i$ and $\overline{\Delta}$. And, by (6), $V_F(0,\overline{\Delta})$ increases for all $\overline{\Delta}$. Thus $\overline{\Delta}^*$ is weakly decreasing in $p$ and $a_I$.

Part (ii). As $\xi$ increases, $V_I(i)$ increases for all $i$. By part (ii) of Lemma 1, $\mu(i,\overline{\Delta})$ increases for all $i$ and $\overline{\Delta}$. Combining with the fact that $\lambda^{\overline{\Delta}} - \xi$ decreases, by (6) we conclude that $V_F(0,\overline{\Delta})$ decreases for all $\overline{\Delta}$. Thus $\overline{\Delta}^*$ is weakly increasing in $\xi$.

Part (iii). As $\lambda$ increases, by (3), $\mu(i,\overline{\Delta})$ remains the same for all $i$ and $\overline{\Delta}$. Since $\lambda^{\overline{\Delta}} - \xi$ increases, by (6), $V_F(0,\overline{\Delta})$ increases for all $\overline{\Delta}$. Thus $\overline{\Delta}^*$ is weakly decreasing in $\lambda$.

Part (iv). As $N$ increases, by part (ii) of Lemma 1, $\mu(i, \overline{\Delta})$ increases for all $i$ and $\overline{\Delta}$. By (6) we conclude that $V_F(0, \overline{\Delta})$ decreases for all $\overline{\Delta}$. Thus $\overline{\Delta}^*$ is weakly increasing in $N$.

Part (v). We first show the following property: $M_S \frac{\partial \mu(i)}{\partial M_S} < \mu(i)$ for all $i$. In particular,

$$\begin{aligned}
\mu(i) - M_S \frac{\partial \mu(i)}{\partial M_S} &\propto \mu_I(i) - V_I(i) \frac{(N-1)(1-p)\mu_I(i) + \rho}{2a_I(2N-1)\mu_I(i) + 2a_I \frac{\rho}{1-p} - (N-1)(1-p)V_I(i)} \\
&\propto a_I(2N-1)\mu_I(i) - (N-1)(1-p)V_I(i) > 0,
\end{aligned}$$

where the second equality uses equation (3), and the inequality uses the result that $\mu_I(i) > \frac{(N-1)(1-p)V_I(i)}{(2N-1)a_I}$ in part (i) of Lemma 1.

Using the property that $M_S \frac{\partial \mu(i)}{\partial M_S} < \mu(i)$ for all $i$, and collecting terms, we have

$$\begin{aligned}
\frac{\partial V_F(0)}{\partial M_S} > & \sum_{z=0}^{\overline{\Delta}-1} \prod_{t=0}^{z} \frac{\iota}{\rho + (1-p)\mu(t) + \iota} \{ \frac{1}{\rho + (1-p)\mu(z) + \iota} \\
& - \sum_{j=z}^{\overline{\Delta}-1} \frac{(1-p)\mu(z)}{[\rho + (1-p)\mu(z) + \iota]^2} \prod_{k=z+1}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} \}.
\end{aligned}$$

Note that the RHS of the above inequality has $\overline{\Delta} - 1$ terms. It is sufficient to show that for every $z$, the term in the bracket is positive. We will only show it holds for $z = 0$, since the proof for other $z$ is similar. More specifically, we want to show

$$\frac{1}{\rho + (1-p)\mu(0) + \iota} - \sum_{j=0}^{\overline{\Delta}-1} \frac{(1-p)\mu(0)}{[\rho + (1-p)\mu(0) + \iota]^2} \prod_{k=1}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} > 0. \tag{12}$$

To show inequality (12) holds, we proceed recursively. The LHS of (12) is proportional to

$$\begin{aligned}
& [\rho + (1-p)\mu(0) + \iota] - \sum_{j=0}^{\overline{\Delta}-1} (1-p)\mu(0) \prod_{k=1}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} \\
= \;& (\rho + \iota) - (1-p)\mu(0) \sum_{j=1}^{\overline{\Delta}-1} \prod_{k=1}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} \\
\propto \;& (\rho + \iota)[\rho + (1-p)\mu(1) + \iota] - \iota(1-p)\mu(0) - \iota(1-p)\mu(0) \sum_{j=2}^{\overline{\Delta}-1} \prod_{k=2}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} \\
> \;& \iota(\rho + \iota) - \iota(1-p)\mu(0) \sum_{j=2}^{\overline{\Delta}-1} \prod_{k=2}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota} \\
\propto \;& (\rho + \iota) - (1-p)\mu(0) \sum_{j=2}^{\overline{\Delta}-1} \prod_{k=2}^{j} \frac{\iota}{\rho + (1-p)\mu(k) + \iota}.
\end{aligned} \tag{13}$$

In the above derivation, the inequality uses Condition 1, $\rho + \iota > \lambda\iota$, and the property in part (i) of Lemma 2, $\mu(0) \leq \lambda\mu(i)$ for all $i \leq \overline{\Delta} - 1$. Repeat the same recursive procedure as in the above derivation, we can show that (13) is bigger than 0. ■

**Proof of Proposition 3.**

**Proof.** We want to show that, for $p \in P(\overline{\Delta}^*)$, $W(p)$ is strictly decreasing in $p$. Observing (9), it is sufficient to show that the last term in the bracket of (9) is strictly increasing in $p$. Let $K(i) \equiv \frac{M_s}{\rho}[1 - (\frac{\iota}{\rho+\iota})^{\overline{\Delta}^* - i}]$. Since $(1-p)\mu(i)$ is decreasing in $p$, it is enough to show that, for all $j$ between 0 and $\overline{\Delta}^* - 1$,

$$Z(j) \equiv \frac{M_s \lambda^{\overline{\Delta}^*} + a_I \mu^2(j)/N + (1-p)\mu(j)K(j)}{\rho + (1-p)\mu(j) + \iota} \tag{14}$$

is increasing in $p$ for all $j$ between 0 and $\overline{\Delta}^* - 1$. Now we take partial derivative of (14) with respect to $p$, which is proportional to:

$$\begin{aligned} \frac{\partial Z(j)}{\partial p} &= [\mu(j) - (1-p)\mu'(j)][M_s \lambda^{\overline{\Delta}^*} - K(j)(\rho + \iota)] + \frac{2a_I}{N}\mu(j)\mu'(j)(\rho+\iota) \qquad (15) \\ &\quad + \frac{a_I \mu^2(j)}{N}[\mu(j) + (1-p)\mu'(j)]. \end{aligned}$$

Note that by previous results, $\mu'(j) < 0$. We first show that $\mu(j) + (1-p)\mu'(j) > 0$, which is equivalent to $\mu_I(j) + (1-p)\mu_I'(j) > 0$. For that purpose, we differentiate equation (3) with respect to $p$:

$$\mu_I'(j) = \frac{-[2a_I \frac{\rho}{(1-p)^2} + (N-1)V_I]\mu_I(j)}{2a_I \frac{\rho}{1-p} - (N-1)(1-p)V_I + 2a_I(2N-1)\mu_I(j)}.$$

Since the numerator is negative, $\mu_I'(j) < 0$ means that the denominator is positive. Using the above equation, we have

$$\begin{aligned} \mu_I(j) + (1-p)\mu_I'(j) &\propto a_I(2N-1)\mu_I^2(j) - (N-1)(1-p)V_I\mu_I(j) \\ &= \rho[V_I - \frac{2a_I}{1-p}\mu_I(j)] > 0, \end{aligned}$$

where the last two steps use equation (3) and the fact that $\mu_I(j) < \frac{(1-p)V_I}{2a_I}$.

Now to show $\frac{\partial Z(j)}{\partial p} > 0$, it is sufficient to show that

$$[\mu(j) - (1-p)\mu'(j)][M_s \lambda^{\overline{\Delta}^*} - K(j)(\rho+\iota)] + \frac{2a_I}{N}\mu(j)\mu'(j)(\rho+\iota) > 0.$$

Again using the fact that $\mu(j) + (1-p)\mu'(j) > 0$, the following inequality is sufficient:

$$-(1-p)\mu'(j)[M_s\lambda^{\overline{\Delta}^*} - K(j)(\rho+\iota)] + \mu(j)[M_s\lambda^{\overline{\Delta}^*} - K(j)(\rho+\iota) - \frac{2a_I}{N(1-p)}\mu(j)(\rho+\iota)] > 0.$$

Now it is enough to show that $M_s\lambda^{\overline{\Delta}^*} - K(j)(\rho+\iota) - \frac{2a_I}{N(1-p)}\mu(j)(\rho+\iota) \geq 0$. Using the fact that $\mu_I(j) < \frac{(1-p)V_I}{2a_I}$, the following inequality is sufficient:

$$\begin{aligned} M_s\lambda^{\overline{\Delta}^*} - [K(j)+V_I](\rho+\iota) &\geq 0 \\ &\Leftrightarrow \lambda^{\overline{\Delta}^*} - \frac{\rho+\iota}{\rho}\xi[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*-j}] \geq 0 \\ &\Longleftarrow \lambda^{\overline{\Delta}^*} - \frac{\rho+\iota}{\rho}\xi[1-(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}] \geq 0. \end{aligned} \tag{16}$$

To show that (16) holds, note that when $\overline{\Delta}^* = 1$, it becomes $\lambda - \xi$, which is positive since $\lambda > \xi > 1$. Now for $\overline{\Delta}^* \geq 2$, the LHS of (16) becomes

$$\lambda^{\overline{\Delta}^*} - \frac{\rho+\iota}{\rho}\xi + \frac{\rho+\iota}{\rho}\xi(\frac{\iota}{\rho+\iota})^{\overline{\Delta}^*}] > \lambda^2 - \frac{\rho+\iota}{\rho}\xi > 0,$$

where the last inequality uses $\lambda > \xi$ and Condition 2 that $\lambda > \frac{\rho+\iota}{\rho}$. ■

**Proof of Proposition 4**.

**Proof.** Let $x \equiv \overline{\Delta}^*$ and $a \equiv \frac{\iota}{\rho+\iota}$. Note that $x$ must be integers and $\lambda a < 1$ by Condition 1. It is sufficient to show that $\frac{(\lambda a)^x}{1-(\lambda a)^x}[1-a^x]$ is strictly decreasing in $x$. For that purpose, we take the difference

$$\begin{aligned} \Pi &\equiv \frac{(\lambda a)^{x+1}}{1-(\lambda a)^{x+1}}[1-a^{x+1}] - \frac{(\lambda a)^x}{1-(\lambda a)^x}[1-a^x] \\ &\propto (\lambda a - 1) + a^x(1-\lambda a^2) + (\lambda a)^{x+1}a^x(a-1) \\ &= (\lambda a - 1)[1-a^x] + a^x(\lambda a)(1-a)[1-(\lambda a)^x]. \end{aligned}$$

By the above expression, $\Pi < 0$ is equivalent to

$$a^x(\lambda a)\frac{1-(\lambda a)^x}{1-\lambda a} < \frac{1-a^x}{1-a} \Leftrightarrow a^x(\lambda a)\sum_{i=0}^{x-1}(\lambda a)^i < \sum_{i=0}^{x-1} a^i.$$

The last inequality obviously holds, since, by $\lambda a < 1$ and $a < 1$, for any $0 \leq i \leq x-1$ we have $a^x(\lambda a)^{i+1} < a^i$. ■

**Proof of Proposition 6**.

**Proof.** Part (i). Let $\lambda' > \lambda$, and superscript $'$ denote the endogenous variables under $\lambda'$. Note that $p^o = \underline{p}(\overline{\Delta}^o)$. By previous analysis, we have $V_F(0, \lambda, p^o, \overline{\Delta}^o) = F$. Since $V_F$ is increasing in $\lambda$ by Proposition 2, $V_F(0, \lambda', p^o, \overline{\Delta}^o) > F$. This implies that $p^{o\prime} < p^o$ as $\overline{\Delta}^o$ does not change. It follows that $\mu'(i) > \mu(i)$ for all $i \leq \overline{\Delta}^o - 1$ by Lemma 1.

Part (ii). Let $\xi' > \xi$, and superscript $'$ denote the endogenous variables under $\xi'$. By previous analysis, we have $V_F(0, \xi, p^o, \overline{\Delta}^o) = F$. Since $V_F$ is decreasing in $\xi$ by Proposition 2, $V_F(0, \xi', p^o, \overline{\Delta}^o) < F$. This implies that $p^{o\prime} > p^o$ as $\overline{\Delta}^o$ does not change.

Part (iii). Let $M_S' > M_S$, and superscript $'$ denote the endogenous variables under $M_S'$. By previous analysis, we have $V_F(0, M_S, p^o, \overline{\Delta}^o) = F$. Since $V_F$ is increasing in $M_S$ by Proposition

2, $V_F(0, M_S', p^o, \overline{\Delta}^o) > F$. This implies that $p^{o\prime} < p^o$ as $\overline{\Delta}^o$ does not change. The fact that $p^{o\prime} < p^o$ and $V_I' > V_I$ (implied by $M_S' > M_S$) means that $\mu'(i) > \mu(i)$ for all $i \leq \overline{\Delta}^o - 1$ by Lemma 1.

Part (iv). Let $N' > N$, and superscript $'$ denote the endogenous variables under $N'$. By previous analysis, we have $V_F(0, N, p^o, \overline{\Delta}^o) = F$. Since $V_F$ is decreasing in $N$ by Proposition 2, $V_F(0, N', p^o, \overline{\Delta}^o) < F$. This implies that $p^{o\prime} > p^o$ as $\overline{\Delta}^o$ does not change. ■

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