Lecture 4
Interest Rates

corrected 10/6 PM as indicated in red

- Interest Rate Mechanics
  - M&B 6
- Real vs. Nominal Rates
  - M&B 8, M&I 4.1
- Loanable Funds Model
  - M&B 19, pp. 1-3
- Debtor-Creditor Redistribution
  - M&I 7.1

Present Value (PV)

\$PV now invested for *m* years at interest rate *i* has Future Value (FV):

$$FV = PV\,(1 + i)^m$$

$\Rightarrow$ Future payment of \$FV to be paid in *m* yrs has PV:

$$PV = \frac{FV}{(1+i)^m}$$

E.g., FV = \$100, *i* = 5% (.05), *m* = 1 yr:

PV = \$100 / (1.05) = \$95.24.

or, if *m* = 20 yr,

PV = \$100 / $(1.05)^{20}$ = \$100 / 2.6533 = \$37.69.

Need x^y or ^ key on calculator to compute!

Note: nominal interest rate *i* is "*R*" in M&B, M&I.

- $\text{PV} = \dfrac{\text{FV}}{(1+i)^m}$ implies that holding FV constant,

$$i\uparrow \rightarrow \text{PV}\downarrow, \quad i\downarrow \rightarrow \text{PV}\uparrow$$

- Also, effect of Δi on PV grows stronger with *m*:

$$\Delta\text{PV/PV} \approx -\Delta i \cdot m$$

  – *Examples:*

    m = 1 yr, i rises from 5% to 6%, Δ i = +1%, FV = 100:

    - Δi · m = - (+1%)(1yr) = -1%

    Actual ΔPV/PV = (94.34 – 95.24)/95.24 = - .0094 = - 0.94%

    m = 10 yrs, i rises from 5% to 6%:

    - Δi · m = - (+1%)(10YR) = -10%

    Actual ΔPV/PV – (55.84 – 61.39)/61.39 = - .090 = - 9.0% (corrected)

  – Leads to *Interest Rate Risk* when banks borrow short, lend long.

i from FV / PV:

$PV = FV / (1+i)^m \Rightarrow$

$(1+i)^m = FV / PV$,

$1+i = (FV / PV)^{1/m}$, so

$$i = (FV / PV)^{1/m} - 1$$

E.g., FV = \$100, PV = \$50, m = 10 yrs.,

$i = (100 / 50)^{1/10} - 1$

$= 2^{0.1} - 1 = 1.0718 - 1 = 0.0718 = \underline{7.18\%}$.

Note: 0.01% = one “Basis Point”.

Bonds

Face Value \$F to be paid at maturity *m*

Coupons \$C paid each year for *m* years.

(Assume annual for simplicity)

Bond Present Value (PV_B)

$$PV_B = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C+F}{(1+i)^m}$$

$$\Rightarrow \; i \uparrow \rightarrow PV_B \downarrow, \quad i \downarrow \rightarrow PV_B \uparrow$$

Bond Duration

Effect of Δi on PV_B again stronger, the longer its m. However, now,

$$\Delta PV_B / PV_B \approx \Delta i \cdot D,$$

where the bond's *Duration D* equals the present-value-weighted average maturity of its payments:

$$D = \left(\frac{(1)C}{(1+i)} + \frac{(2)C}{(1+i)^2} + \ldots + \frac{(m)(C+F)}{(1+i)^m} \right) / PV_B$$

Generally,

D = m if C = 0,

D < m if C > 0,

D increases with m

Yield to Maturity (YTM)

= the value of *i* that gives back market price of bond, holding C, F, m constant.

If

$PV_B = F$, bond is "at par", $YTM = C / F$

$PV_B > F$, bond is "above par", $YTM < C / F$

$PV_B < F$, bond is "below par", $YTM > C / F$

E.g.

$F = \$100$, $C = \$4$, $PV_B = \$100 \Rightarrow YTM = 4\%$

$F = \$100$, $C = \$3$, $PV_B = \$110 \Rightarrow YTM < 3\%$

$F = \$100$, $C = \$6$, $PV_B = \$90 \Rightarrow YTM > 6\%$ (corrected)

Consols (Perpetuities)

Pay \$C / yr. forever

Exist in UK, conceptually important

$$\mathrm{PV_C} = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \quad \ldots \quad \infty$$

$$= \frac{C}{1+i} + \frac{1}{(1+i)}\left[\frac{C}{1+i} + \frac{C}{(1+i)^2} + \quad \ldots \quad \infty\right]$$

$$= \frac{C}{1+i} + \frac{1}{(1+i)}\left[\mathrm{PV_C}\right]$$

$\Rightarrow$ $(1+i)\,\mathrm{PV_C} = \mathrm{C} + \mathrm{PV_C}$,

$\mathrm{PV_C} = \mathrm{C} / \mathrm{i}$

E.g., C = \$100, i = 4% $\Rightarrow$ $\mathrm{PV_C}$ = 100/.04 = \$2500

Real vs. Nominal Interest Rates

i = *nominal* interest rate

on $-denominated loans, not indexed for inflation.

r = *real* interest rate

on purchasing-power-denominated loans, with payments indexed for inflation

Note: nominal interest rate *i* is “*R*” in M&B, M&I.

US Treasury Inflation-Protection Securities (TIPS)

- All payments indexed to CPI-U (2.5 mo. lag)
- Provide direct observation of real rate *r*
- First issued Jan. 1997
- 5, 10, 20, 30-year initial maturities
  - longest now 24 yrs.
- Now $472 B (10.4% of marketable Treasury debt)

❖ Monthly TIPS yield curves on my webpage:

www.econ.ohio-state.edu/jhm/ts/ts.html

Present Values with P-indexed loans

Same formulas, with *r* in place of *i*

$PV = FV / (1+r)^m$, etc.

e.g. Indexed Consol:

C = *Real* coupon payment (today's $)

$PV_C = C / r$ (today's $)

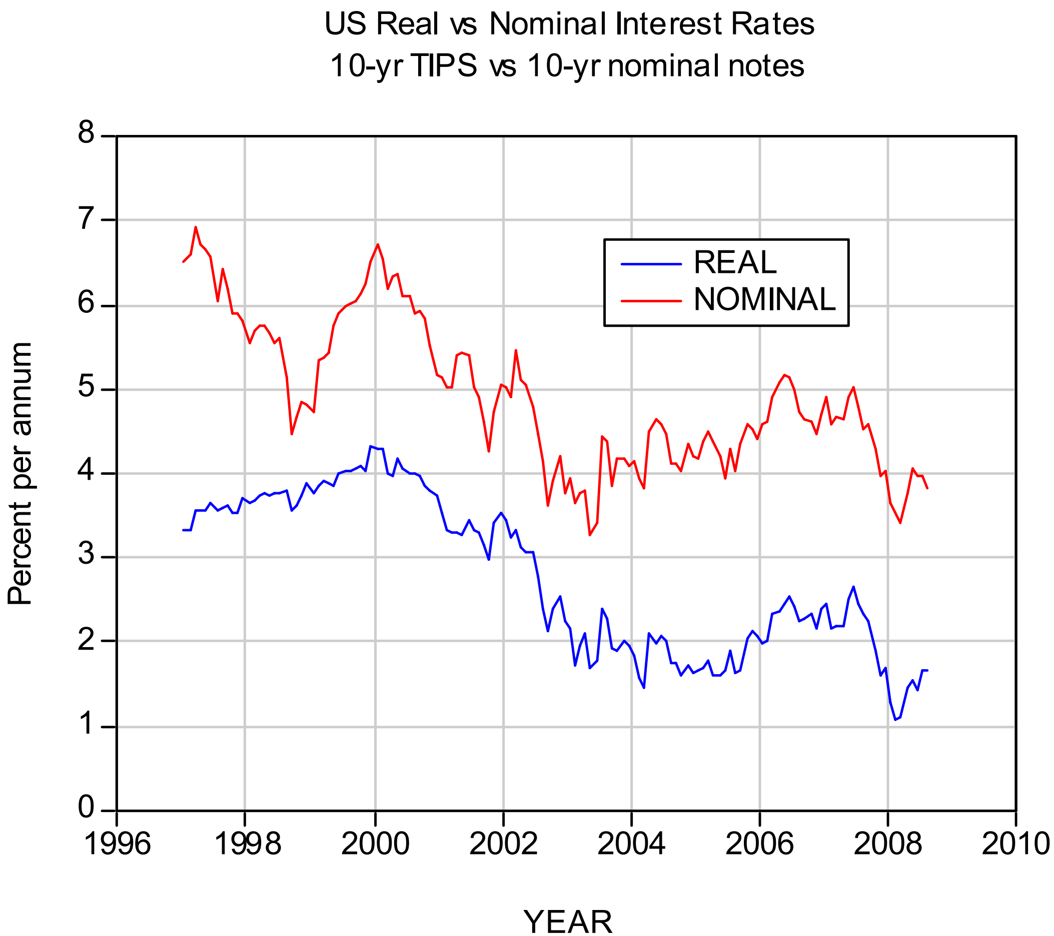
application:

Parcel now pays $10,000 rent per year, future rent will grow in proportion to P. *r* = 2%.

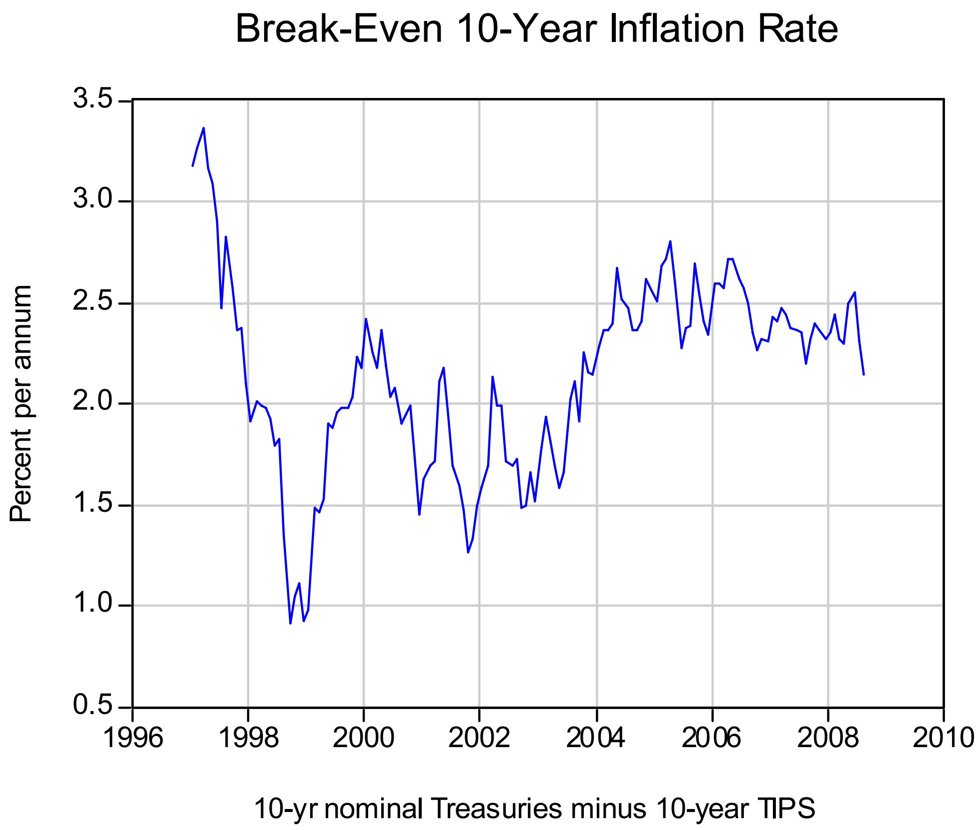
$\Rightarrow$ PV = $10,000 / .02 = <u>$500,000</u>

- Real YTM on 10-yr TIPS have been between 1% and 4.5% since their introduction in 1997.

- Nominal YTM on conventional Treasuries higher, to compensate for likely future inflation. Also more volatile, because of fluctuations in inflation premium.



i minus *r* gives "Break-Even" inflation rate, at which returns on real and nominal bonds are equal.



Determination of r, i

- Loanable Funds Model

  r primarily determined by savings, investment decisions

- Fisher Equation

  $$i = r + \pi^e,$$

  where π^e is *expected inflation* over life of loan

- Adaptive Learning (AL)

  π^e mostly determined by past π, with biggest weights on recent past.

  Coefficients may change over time.

• Before 1997, *r* not directly observed.

• But AL allows us to infer r from i, using Fisher Eq'n:

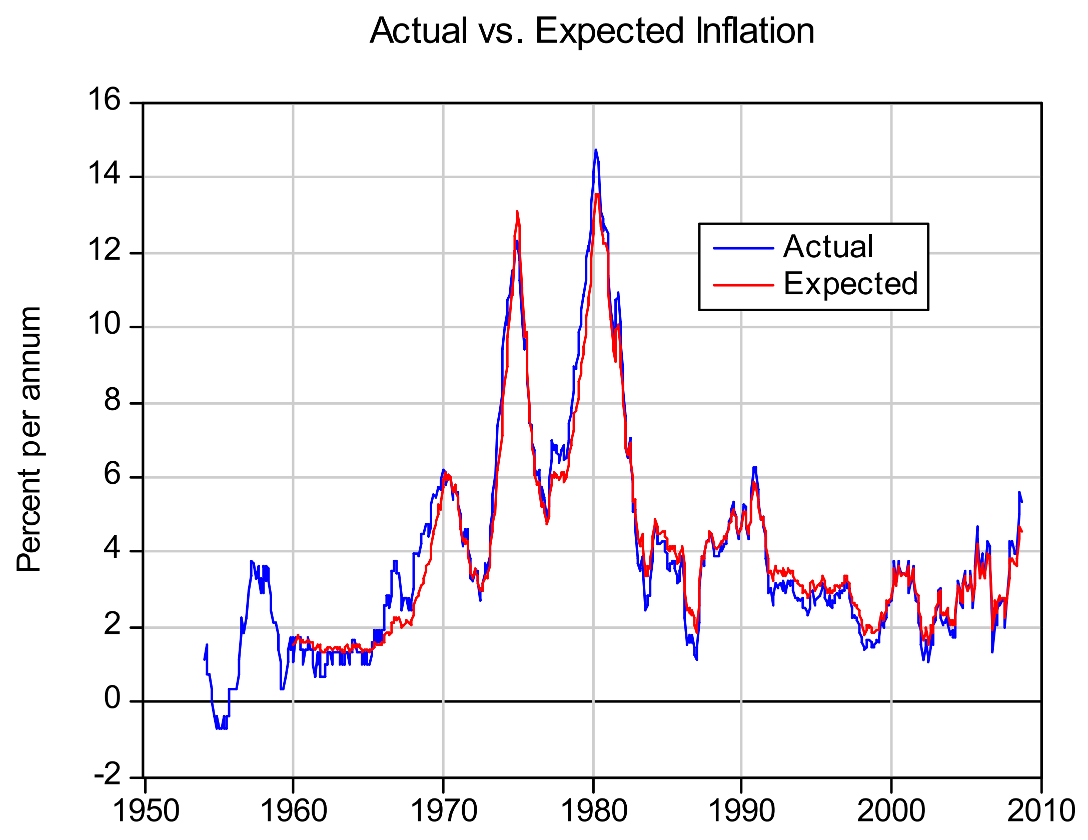
$$r = i - \pi^e$$

plus recent inflation.

• Currently, for 1-yr horizon,

$$\pi^e = 1.24 + .64\ \pi$$

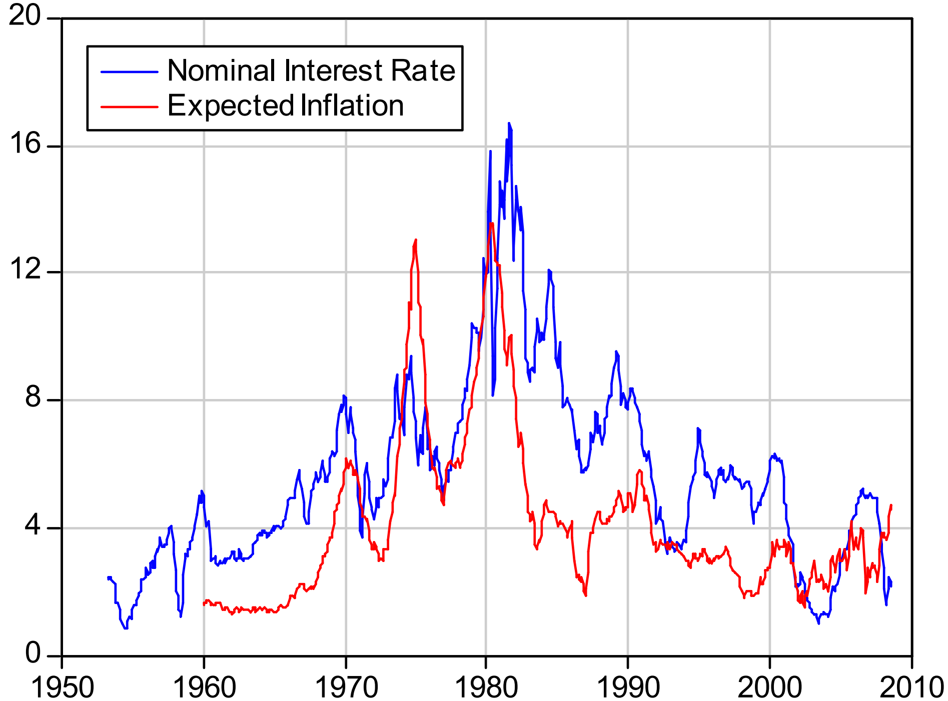
where π is avg. inflation over past 12 mo.



Actual year-over-year past inflation,
Adaptive Lag estimate of expected inflation for coming year

Changes in π^e account for much of the movement in nominal rates over past 50 years ...

Nominal Interest Rate vs Expected Inflation



One-year const. mat. nominal US Treasury yield,
Adaptive Lag estimate of 1-year inflation expectations

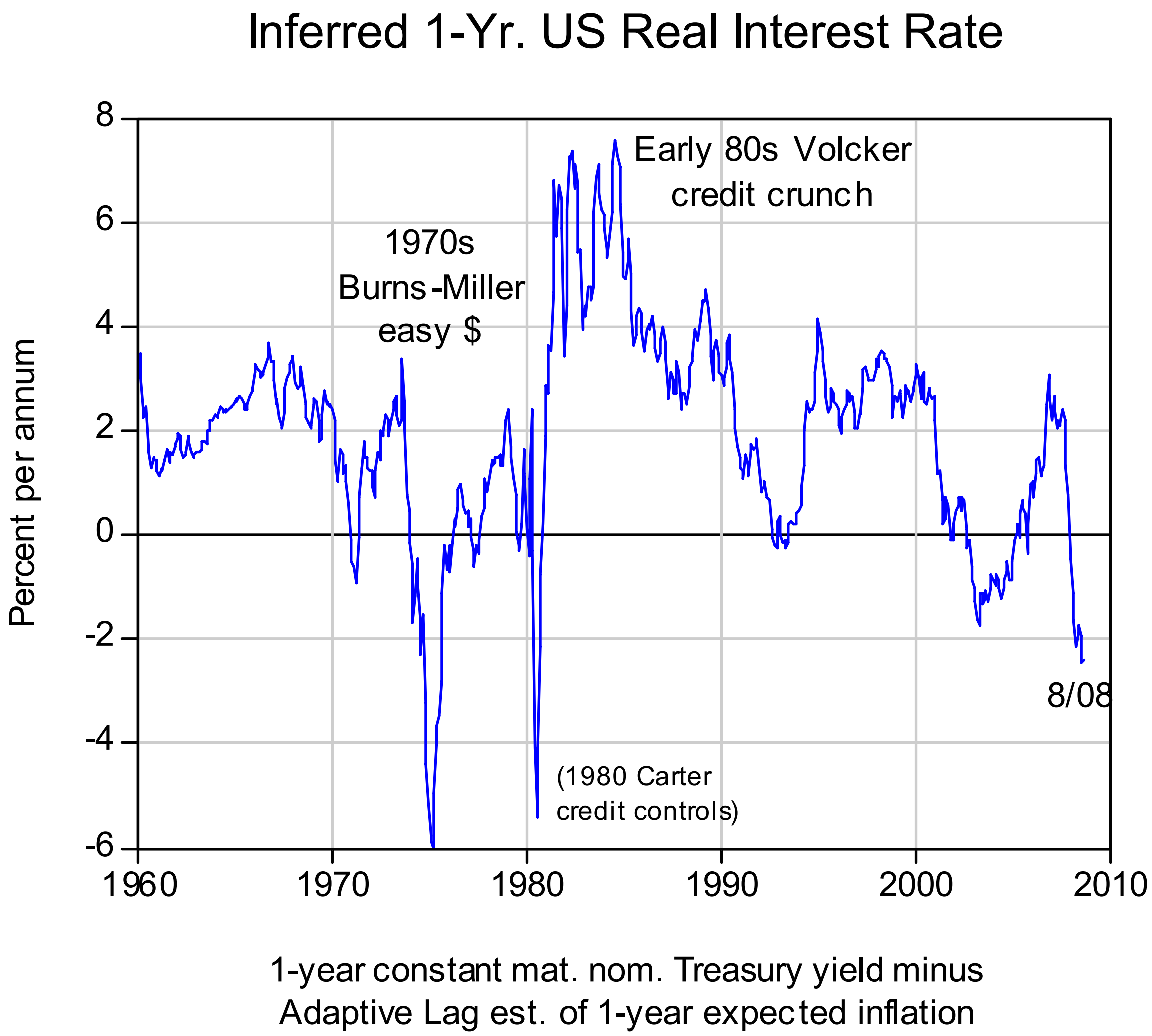
... but inferred real rates have not been constant:

1-yr *r* typically about 2%,

but was 0-1% in 1970s,

5-6% in early 80s,

negative 2003-5, 2008.



Loanable Funds Model of *r* (M&I 19, pp. 1-3)

$(1+r)^m$ is price of present goods in terms of future goods

- $r\uparrow \Rightarrow$ present goods more costly (rel. to future goods)
- $r\downarrow \Rightarrow$ present goods less costly.

"Credit" = command over present goods

= what you get in exchange for your IOU when you borrow

= what you give up in exchange for someone else's IOU when you lend.

Non-monetary equilibrium *r* determined by

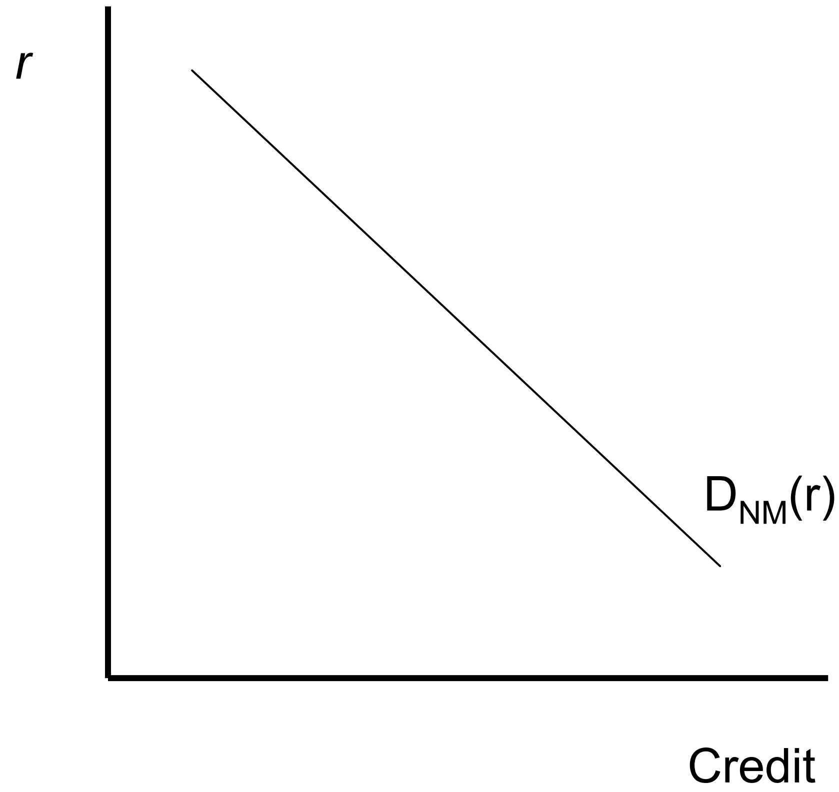
Demand & Supply of Credit.

Non-Monetary Demand for Credit by Borrowers, $D_{NM}(r)$

At low r, borrowers want more credit.

At high r, borrowers want less credit.

$\Rightarrow$ $D_{NM}(r)$ slopes down.

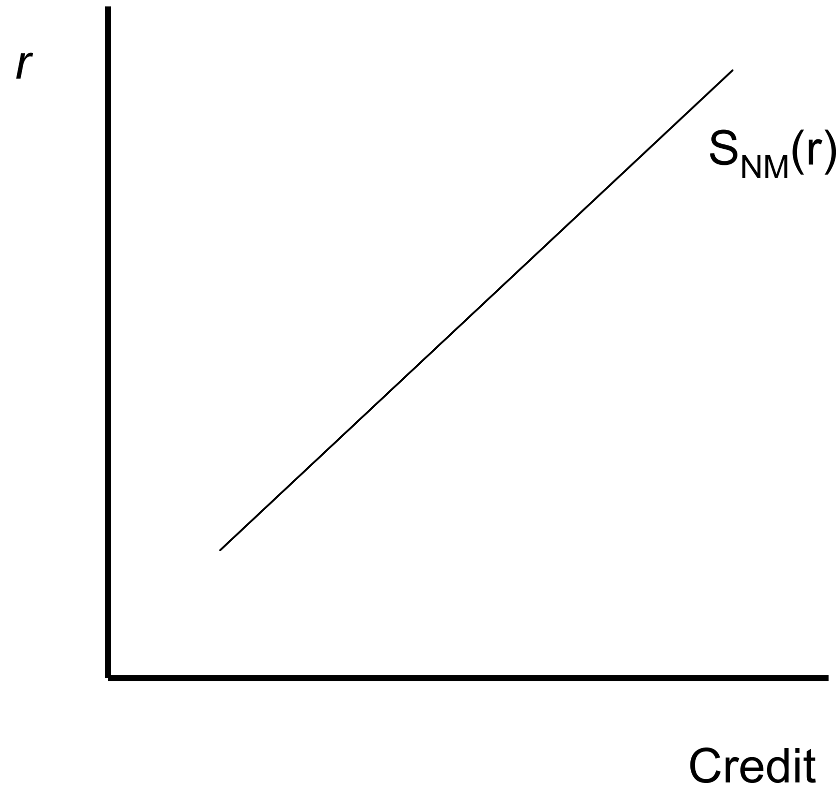


Non-Monetary Supply of Credit by Lenders

At high r, lenders willing to give up more credit

At low r, lenders give up less credit.

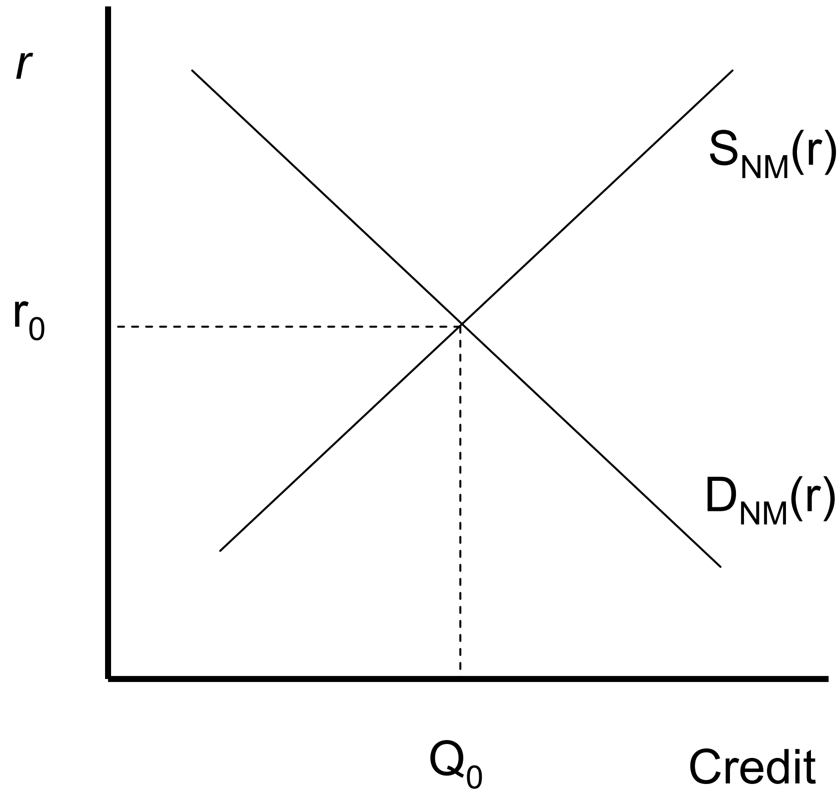
⇒ $S_{NM}(r)$ slopes up



Credit Market Equilibrium

(non-monetary economy)

r_0 = Non-Monetary Equilibrium real interest rate



Debtor-Creditor redistribution (M&I 7.1)

- Nominal Debt paying $i = r + \pi^e$

1. $\pi = \pi^e$

   $\Rightarrow i - \pi = r$, No transfer.

2. $\pi > \pi^e$ (as in 1970s)

   $\Rightarrow i - \pi < r$. Creditors lose, Debtors gain.

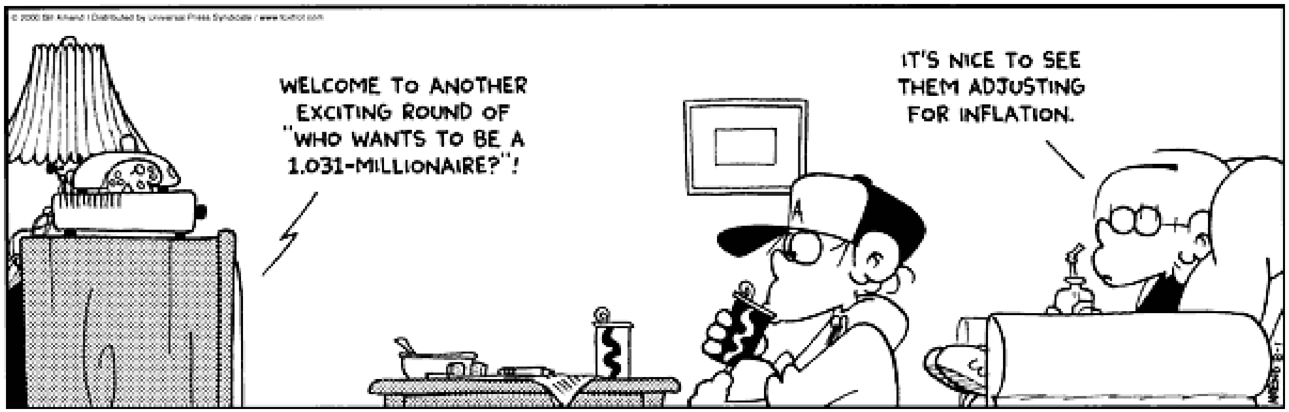
3. $\pi < \pi^e$ (1930's, 1980's)

   $\Rightarrow i - \pi > r$. Debtors lose, Creditors gain.*

* if they can collect – Bankruptcies & foreclosures rise!

- Transfer may be eliminated with Price-Level Indexed Debt.
  - Payments indexed to CPI-U or other index
  - Real return independent of inflation
  - TIPS since 1997
- Nominal debt = safe indexed debt + lottery ticket on CPI.
  - Serves no function for risk-averse investors, borrowers
  - But still no private indexed securities to speak of!

FOXTROT **by Bill Amend**



- Next:
  - – Velocity and the Quantity Equation
    - M&I 3, 4, 7.4