

⑨ Missing Pages

Durbin - Watson Test for $\rho_1 = 0$:

$$DW = \frac{\sum_{t=2}^T (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2}{\sum_{t=1}^T (\hat{\epsilon}_t^2)}$$

$$= \frac{\sum_{t=2}^T \hat{\epsilon}_t^2 - 2 \sum_{t=2}^T \hat{\epsilon}_t \hat{\epsilon}_{t-1} + \sum_{t=2}^T \hat{\epsilon}_{t-1}^2}{\sum_{t=1}^T \hat{\epsilon}_t^2}$$

$$= \frac{\sum_{t=2}^T \hat{\epsilon}_t^2 + \sum_{t=1}^{T-1} \hat{\epsilon}_t^2}{\sum_{t=1}^T \hat{\epsilon}_t^2} - \frac{2(T-2) \hat{Cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1})}{(T-1) \hat{Var}(\hat{\epsilon}_t)}$$

$$\approx 2 \frac{T-1}{T} - 2 \frac{T-2}{T-1} \hat{\rho}_1$$

$$\rightarrow \underline{2(1 - \hat{\rho}_1)} \text{ as } T \rightarrow \infty$$

Properties:

- $0 \leq DW \leq 4$
- $DW \approx 2$ if $\hat{\rho}_1 = 0$
- $DW < 2$ if $\hat{\rho}_1 > 0$ (common)
- $DW > 2$ if $\hat{\rho}_1 < 0$ (rare)
- Automatically reported by EViews, most others.

(1-Tailed)

TABLE 5
FIVE PERCENT SIGNIFICANCE POINTS OF d_l AND d_u FOR DURBIN-WATSON TEST†

N	k = 1		k = 2		k = 3		k = 4		k = 5	
	d_l	d_u	d_l	d_u	d_l	d_u	d_l	d_u	d_l	d_u
15	1.08	1.36	.95	1.54	.82	1.75	.69	1.97	.56	2.21
16	1.10	1.37	.98	1.54	.86	1.73	.74	1.93	.62	2.15
17	1.13	1.38	1.02	1.54	.90	1.71	.78	1.90	.67	2.10
18	1.16	1.39	1.05	1.53	.93	1.69	.82	1.87	.71	2.06
19	1.18	1.40	1.08	1.53	.97	1.68	.86	1.85	.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	.90	1.83	.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	.93	1.81	.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	.96	1.80	.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	.99	1.79	.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.53	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

† N = number of observations; k = number of explanatory variables (excluding the constant term).
Source: Reprinted with permission from J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression," *Biometrika*, vol. 38, pp. 159-177, 1951.

$H_0: \rho_1 = 0$ $H_1: \rho_1 > 0$ (1-Tailed!)

"k" excludes const. (as in Wooldridge)

from Pindyck + Rubinfeld text.

Exact Dist. of DW depends on T,
of regressors, serial correlation of
regressors.

Exact critical values, p-values computable
but not in EViews.

(In SHAZAM, use OLS/DW)

Table gives upper + lower bounds d_l, d_u

@ 5%, 1-tailed test. $0 < d_l < d_u < 2$

$H_0: \rho_1 = 0 \quad H_1: \rho_1 > 0$

- $DW < d_l$ Reject H_0 @ 5%
- $d_l < DW < d_u$ Inconclusive - use exact p-value if avail.
- $d_u < DW$ Accept H_0

"K" of Table

= # regressors excl. C.

As $T \rightarrow \infty$,

$$\hat{\rho}_2 \rightarrow N\left(0, \frac{1}{T}\right) \quad \text{if } \rho_2 = 0$$

regardless of K , behavior of regressors.

$$\Rightarrow \underline{DW} \approx 2(1 - \hat{\rho}_1) \rightarrow N\left(2, \frac{4}{T}\right)$$

$$\text{or } \underline{\sqrt{T} \left(1 - \frac{DW}{2}\right)} \sim N(0, 1)$$
$$\approx \sqrt{T} \hat{\rho}_1$$

Finite Sample

eg $T = 100$

From Normal Approx., (1-tailed)

$$DW^{crit}(.05) \approx 2 - \frac{2}{\sqrt{100}} (1.645)$$
$$= \underline{1.67}$$

From Table 5,

K	d_e	d_u
1	<u>1.65</u>	<u>1.69</u>
5	<u>1.57</u>	<u>1.78</u>

(excl. const.)

Correlogram

- Plots $\hat{\rho}_L$ vs L (lag L)
 - with $N(0, 1/n)$ 95% c.i. (dotted)
- Tabulated under "AC"
- Gives Box-Ljung Q-Stat (Econ 642)
 - with p-value
 - Q_L tests $\rho_1 = \rho_2 = \dots = \rho_L = 0$
 - $\rightarrow \chi^2_L$ as $n \rightarrow \infty$

• EViews

Equation window

VIEW > Residual Tests
> Correlogram

(also gives Partial Autocorrelation (PAC)
- Econ 642)

basedata.txt

1959Q1	Year	Non-Base	P	Real GDP	3 mo-T-Bill rate
	yr	Base		y	R
1959.00	1959.00	40.548	20.680	2392.886	2.773
1959.25	1959.25	40.725	20.711	2455.813	3.000
1959.50	1959.50	40.974	20.770	2453.948	3.540
1959.75	1959.75	40.868	20.853	2462.587	4.230
1960.00	1960.00	40.692	20.903	2517.365	3.873
1960.25	1960.25	40.650	20.995	2504.800	2.993
1960.50	1960.50	40.910	21.093	2508.726	2.360
1960.75	1960.75	41.068	21.186	2476.232	2.307
1961.00	1961.00	40.919	21.210	2491.154	2.350
1961.25	1961.25	40.839	21.249	2537.981	2.303
1961.50	1961.50	41.208	21.305	2579.114	2.303
1961.75	1961.75	41.762	21.360	2631.831	2.460
1962.00	1962.00	41.866	21.482	2679.149	2.723
1962.25	1962.25	42.183	21.538	2708.404	2.717
1962.50	1962.50	42.475	21.596	2733.339	2.840
1962.75	1962.75	42.839	21.671	2740.014	2.813
1963.00	1963.00	43.151	21.732	2775.944	2.907
1963.25	1963.25	43.603	21.754	2810.598	2.940
1963.50	1963.50	44.182	21.794	2863.512	3.293
1963.75	1963.75	44.730	21.923	2885.796	3.497
1964.00	1964.00	45.175	22.001	2950.490	3.530
1964.25	1964.25	45.693	22.073	2984.751	3.477
1964.50	1964.50	46.422	22.180	3025.492	3.497
1964.75	1964.75	47.030	22.282	3033.640	3.683
1965.00	1965.00	47.455	22.380	3108.151	3.890
1965.25	1965.25	47.860	22.479	3150.167	3.873
1965.50	1965.50	48.482	22.578	3214.076	3.867
1965.75	1965.75	49.294	22.717	3291.826	4.167
1966.00	1966.00	50.025	22.857	3372.325	4.610
1966.25	1966.25	50.595	23.071	3384.007	4.587
1966.50	1966.50	51.054	23.293	3406.292	5.043
1966.75	1966.75	51.396	23.498	3433.681	5.210
1967.00	1967.00	52.181	23.611	3464.114	4.513
1967.25	1967.25	52.689	23.759	3464.310	3.660
1967.50	1967.50	53.394	23.977	3491.798	4.300
1967.75	1967.75	54.288	24.242	3518.205	4.753
1968.00	1968.00	55.166	24.503	3590.655	5.050
1968.25	1968.25	55.962	24.777	3651.618	5.520
1968.50	1968.50	56.921	25.017	3676.455	5.197
1968.75	1968.75	57.907	25.367	3691.966	5.587
1969.00	1969.00	58.838	25.622	3750.180	6.093
1969.25	1969.25	59.469	25.966	3760.881	6.197
1969.50	1969.50	60.069	26.345	3784.245	7.023
1969.75	1969.75	61.144	26.678	3766.280	7.353
1970.00	1970.00	61.976	27.051	3759.997	7.210
1970.25	1970.25	62.940	27.437	3767.066	6.677
1970.50	1970.50	63.752	27.655	3800.541	6.327
1970.75	1970.75	64.630	28.009	3759.801	5.353
1971.00	1971.00	65.987	28.429	3864.057	3.840
2008	2008.75	~	NaN	NaN	~

5th. Leno Fed

FRED

Outlier