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TRADEMARK SALES, ENTRY, AND THE VALUE OF REPUTATION*

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We develop an infinite-horizon, overlapping-generations model of reputation in which consumers base willingness to pay for agent services on past performance summarized by a trademark. We show that when trademarks can be sold, successful firms capture the full value of their reputations upon sale but receive smaller premia for good performance while active as service providers. With discounting, all agents are worse off with trademark trade. Taking entry cost into account, we show that trademark trade typically reduces entry. When entry costs are high, welfare is increased by prohibiting such trade.

1. INTRODUCTION

A trademark is a label, whether word, symbol, sound, color, or other signifier (signifiant), used by a person to distinguish his or her goods or services from those sold by others.[2] Trademarks are thus the legal form of names that serve as carriers of reputation. The legal protection afforded to trademarks has long been focused on preventing consumer confusion concerning the link between a trademark and the underlying performance that established the reputation that the mark represents. But more recently, legal protection has been extended to prevent trademark "dilution," defined to be "the lessening of the capacity of a famous mark to identify and distinguish goods or services, regardless of the presence or absence of . . . likelihood of confusion, mistake, or deception."[3] Despite

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[2] Trademarks are defined as part of the Lanham Act, 15 U.S.C. §1125, the Federal statute governing their protection and use. Trademarks are also protected under common law and by statutes adopted by states.

Trademarks formally include only marks that identify goods, but for purposes of this paper we use the term "trademark," which we sometimes shorten to "mark," to denote legal names that identify and distinguish both goods and services, thus including legal trademarks, service marks, certification marks, and collective marks.

[3] Dilution entered federal statute law with the passage of the Federal Trademark Dilution Act of 1995 (FTDA), Pub. L. No. 104–98 (1996). With passage of this statute, Congress joined more than one half of the states in providing statutory protection against trademark dilution.

considerable litigation, the meaning of dilution as distinct from confusion remains unclear. This article develops a model of trademarks as reputations in which there is no possibility of confusion between a trademark and the underlying history of provision of the good or service to which that trademark attaches. But although the link between a given trademark and past performance associated with that mark is unambiguous, we identify circumstances under which trademark dilution occurs nonetheless. We thus provide what we believe to be the first formal model to analyze the trademark dilution problem.

The insertion into trademark law of dilution as a cause of action has been a central part of a broader and very controversial move toward the "propertization" of trademarks.[4] Once trademarks have been accorded the status of property possessing intrinsic value, it is natural to suppose that their owners will be free to sell them. Under United States law, however, the sale or license of trademarks is prohibited "except as an incident to selling or licensing the right to produce the good that the mark identifies" (Landes and Posner, 2003, p. 184). But the U.S. ban on "naked" trademark sales (known as "assignments in gross") contrasts starkly with the GATT agreement on Trade-Related Aspects of Intellectual Property Rights[5] (the "TRIPs" agreement) to which the United States has subscribed. Indeed, that agreement expressly gives a trademark owner "the right to assign the trademark with or without the transfer of the business to which the trademark belongs."[6]

Our framework permits us to assess the effects of these opposing legal standards. Since more able agents are most likely to accumulate successful trademarks, it seems reasonable to suppose that allowing the sale of trademarks will encourage entry by providers of high ability, increasing welfare both by inducing better agents and by conserving the value of trademarks. Modeling the acquisition and sale of reputations in an overlapping generations model, we show that the above intuition regarding trademark sales is incorrect. Permitting successful agents to sell their reputations (trademarks) does indeed provide the prospect of higher benefits to agents as they consider entry into the marketplace, but that benefit comes at the cost of dilution of the value that reputations generate during the working lives of successful agents.

More formally, we provide a complete characterization of what we believe to be the most natural equilibrium of our overlapping-generations framework. High quality agents in our equilibrium are more likely to be able to sell their trademarks upon retirement, but trade also causes them the prospect of lower returns to their

The FTDA added the definition of dilution to the United States Code, where it appears as 15 U.S.C. §1127.

4 Lemley (1999, p. 1693) provides an example: "There is an increasing tendency to treat trademarks as assets with their own intrinsic value, rather than as a means to an end." Lemley's decision to characterize the expansion of trademark law as "the death of common sense" testifies to the controversy surrounding the issue.

5 33 I.L.M. 81 (December 15, 1993).

6 See Article 21 of the TRIPS Agreement, http://www.wto.org/english/docs_e/legal_e/27-trips_04_e.htm. The North American Free Trade Agreement contains identical language. See Article 1708, Section 11.

reputations during their working lives.[7] These effects exactly offset one another in nominal terms, so that the effect of trade is to postpone returns to good performance until an agent retires, at which point it sells its trademark. With discounting, this postponement reduces the expected lifetime return to agents. Taking agent entry into account, we show that allowing trademark sales reduces entry. The welfare implications of allowing trademark sales is nicely parameterized by the entry cost level. When entry cost is low, equilibrium entry is typically excessive compared to the social optimum. Allowing trademark trade improves welfare by reducing entry. When entry cost is high, equilibrium entry is typically insufficient compared to the social optimum. In this case, a ban on trademark trade improves welfare by encouraging entry of high-quality agents. Our results thus indicate that as long as entry by new agents is viewed as difficult or costly, the otherwise surprising U.S. policy of preventing "naked" trademark sales has a basis in theory. We do not argue that the U.S. rule is always correct, for when agent entry is cheap and easy, entry can be excessive as low-ability agents flood the market.[8] Some of this excess can be stemmed by allowing trademark trade that facilitates dilution. We believe, however, that support for permitting trademark trade arises from the view that such trade encourages more agents to enter. In our model, the reverse occurs.

Our article is a contribution to the growing literature that models firms primarily as the bearers of reputation, which includes, for example, Kreps (1990), Tadelis (1999, 2002, 2003), Mailath and Samuelson (2001), Hörner (2002), Cai and Obara (2004), and Rob and Fishman (2005). In particular, our article is most closely related to Tadelis (1999, 2003), where reputations are modeled within a pure adverse selection framework.[9] By considering consumers who live for only a single period, the Tadelis model ensures that a firm's reputation is the only observable intertemporal linkage.[10] Tadelis demonstrates that when such names can be traded, and when those trades cannot be observed by consumers, available names *will* be traded in all equilibria. Tadelis' model is ideally suited to our purpose in order to represent the case corresponding to the TRIPs rule that trade in trademarks is permitted. Tadelis (1999, p. 548) provides "a model in which a firm's only asset is its name, which summarizes its reputation ... " (see also Tadelis, 2003). Firms in the Tadelis model consist of agents that offer to provide consumers with a service. The names that Tadelis models are, in the rubric of the law, trademarks. He thus shows that when a market in trademarks is permitted, trademarks of successful firms will be traded in all equilibria. Even more important for our purposes is

[7] The characterization of our results given here applies to all generations except the first. Members of the first generation gain no benefit from prior performances, and hence cannot suffer dilution. Trademark trade must benefit them. See Section 3 below for details.

[8] Moreover, our model considers only innate differences in agent ability. The prospect of selling a successful trademark can induce more effort by agents in a moral hazard setting.

[9] The models of Cai and Obara (2004) and Rob and Fishman (2005) are based on pure moral hazard. Models based on both adverse selection and moral hazard include Diamond (1989), Mailath and Samuelson (2001), and Tadelis (2002). For another application of an overlapping generations model in the context of reputation see Cabral's (2000) analysis of brand extension.

[10] See also Diamond (1989).

his additional result that trademarks that label good reputations connoting high-quality agents will be purchased in equilibrium by at least some agents that are of lower quality than the sellers that built and now offer strong trademarks for sale. Both features remain in the equilibrium that we constructed in this article.

There are two major differences between our approach and that of Tadelis. First, given its fixed set of agents, the Tadelis adverse selection model does not permit welfare analysis of the effects of name trade. We introduce costly entry, and thereby model the set of agents active in our market as endogenously determined. By incorporating entry costs, we are able to deduce welfare effects of the legal rules governing such trade. In a companion paper, Tadelis (2002) introduces moral hazard into his adverse selection framework so that agents' ability to provide successful services can be affected by their efforts. The welfare effect with name trade identified in Tadelis (2002) is ambiguous. Our approach is thus complementary to that of Tadelis (2002). Second, the Tadelis approach is based primarily on a finite horizon. Given our objectives of analyzing the effect of trade in trademarks on entry and welfare in the stationary state, we extend the model to the infinite horizon. We construct what we believe to be the first competitive equilibrium with rational expectations in such a model. We specify very carefully how trademarks are assigned to purchasers so that the veil of the trademark is not pierced in such a way as to permit consumers to identify agents other than by past successes. Although we do not attempt to suggest a refinement to select our equilibrium, we provide a set of criteria under which our equilibrium arises naturally.

Section 2 formally introduces our overlapping generations model. Section 3 provides a complete specification of our equilibrium when the set of agents is fixed. We identify a trademark dilution effect based on the equilibrium we constructed. Section 4 introduces entry into the model, showing that trademark trade reduces the measure of agents entering the market. This result permits us to identify unambiguously the welfare consequences of permitting trademark trade as a function of the cost of entry. Section 5 is a discussion of our equilibrium focus and construction. Section 6 summarizes the analysis and offers concluding remarks.

2. THE MODEL

We model agents who are active in the market for a service for two periods and who then retire. Our model considers an infinite number of overlapping cohorts, or generations, of these agents, as illustrated in Figure 1. Time is denoted by $t \in \{1, \ldots, \infty\}$. Each agent offers to provide consumers with a service. The service offered is identical, but agents differ in their innate ability to provide it successfully.[11] A successful outcome as judged by consumers is denoted by S and, correspondingly, an unsuccessful consumer experience (failed service provision)

[11] We define the market for services in prospect, rather than outcome, in the sense that many architects may offer to design a house, and are thus in the same market, even if the resulting designs would vary substantially.

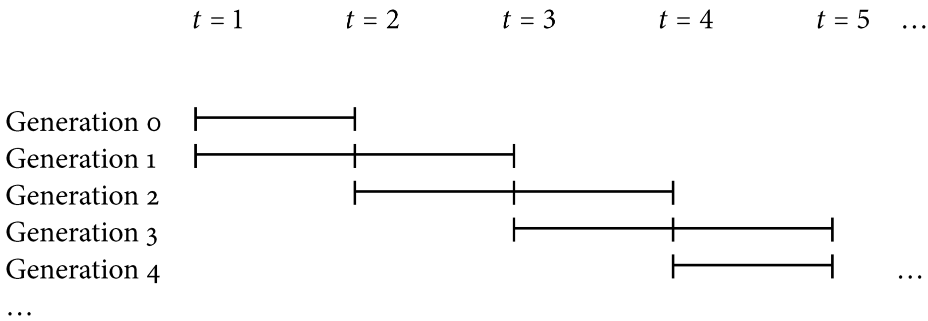

FIGURE 1

AN INFINITE HORIZON OLG ECONOMY

by F.[12] We assume that each agent's type is characterized by its probability of success, $\theta \in [0, 1]$. The measure of the agents in each generation is one. Ex ante, the agents' types are assumed to be uniformly distributed over [0, 1]. As noted above, each cohort is active for two periods. Upon a cohort's retirement, its agents are replaced by a new cohort of the same size. Thus two cohorts are active at any one time.

The service that each agent supplies is valued identically at one by each consumer if that service is provided successfully, and zero if the provision is a failure. The service is an experience good (Nelson, 1970), in that the consumer must contract with an agent to provide the service without being able to inspect the quality of the service the agent is offering in advance. Payments are made when the contract is entered into. Agents discount future payoffs with the common discount factor $\delta \in (0, 1)$.[13] Each consumer has reservation utility of zero and consumes at most the service offered by a single agent. Consumers are active for only one period. We assume that the measure of customers, μ_C, exceeds the measure of agents in the market during any period, $\mu_C > 2$. As long as customers must select from a pool of (indistinguishable to consumers) agents, this assumption guarantees that all agents will be employed.

We assume that sufficient trademarks are available as identifiers to permit each agent to choose a unique unused identifier if it so chooses. At the beginning of each period, each agent acquires a unique identifier, its trademark. At the end of each period, the qualities of the services are realized. Each active agent acquires a history S ("success") or F ("failure") from its just-completed performance. Agents that have chosen new trademarks are denoted by N. At the end of the first period, an active agent must have either an S or an F history. If an agent's history can be erased, the new history becomes N. We assume that at the beginning of each period, each active agent can choose either to retain its past trademark or, at no

[12] All consumers are assumed to agree on their assessment of outcomes.

[13] Our main results depend on strict discounting. Although the full discounting case ($\delta = 0$) makes our infinite horizon analysis uninteresting and hence not considered here, we will discuss the effect of no discounting ($\delta = 1$) in the text.

cost, to select a new trademark.[14] When a new trademark is adopted, the agent is able to shed its history, along with its mark.[15]

If trademarks cannot be bought or sold, this completes the description of our economy. We use this no-trade economy as a base against which we can measure the effects of permitting trademarks to be bought and sold. Our alternative economy adds a market for trademark sales to the underlying market for services. More formally, at the end of each service-provision period, all agents (new, continuing, and the retiring) enter the market for trademarks. Since retiring agents no longer offer services, they will always be on the supply side of the trademark market so long as they possess a valuable trademark. In contrast, new agents can only participate initially on the demand side of the trademark market. Continuing agents who have just completed their first working period may choose to either keep, sell, or abandon the marks they have earned. Each agent, no matter whether it enters the market on the demand or the supply side, may buy or sell as many times as it wishes, subject to the proviso that no agent owns more than one mark at a time.

Comparison of these two economies permits us to assess the impact of trademark trades on the amount of dilution that occurs. We will identify these cases with superscript $k = 0, T$, for the no-trade and tradable trademark cases, respectively.

Just as unsuccessful agents can abandon their marks—and thereby their histories—without cost, we assume that trade in trademarks occurs without the knowledge of consumers. A mark can become associated with an agent that had no part in compiling the record that the mark records. In other words, transfers between agents in the ownerships of marks are assumed to be unobservable by consumers. The entity that the trademark records is thus separate from the identity of the agent that will deliver the service in question.[16]

The results of service provision are assumed to be common knowledge, and become associated with the trademark of the corresponding agent. For example, an S trademark is one that has been in the market for one period, during which the underlying agent provided a successful experience. An SF trademark is one that has been in the market for two periods with realized performance by the

[14] It is clear that new trademarks are easily obtained. The number of new trademarks issued by the United States Patent and Trademark office has exceeded 100,000 annually since 2000 (See United States Patent and Trademark Office, "Performance and Accountability Report for Fiscal Year 2004," `http://www.uspto.gov/web/offices/com/annual/2004/index.html`, Table 18, p. 134.). Moreover, no legal obstacles to switching names exist, and indeed, such switching is commonly practiced by counterfeiters who change names when challenged by reputable trademark holders (private communication with intellectual property attorney). A firm cannot escape liability to past customers for poor payments, but can shed the trademark associated with poor performance as regards prospective consumers.

[15] Note that any history repeating with multiple N's such as NN cannot be distinguished from N by consumers. All such histories are thus denoted simply as N.

[16] In our no-trade benchmark, the reputation of an agent applies only to the history that agent has amassed, thus guaranteeing that mark identifies the agent responsible for the reputation. In contrast, with trademark trade, continuity is broken. Through trademark sale, an agent's history, as recorded under its trademark, may instead record the performance of a different agent.

underlying agent or agents of S followed by F. Thus, every agent can either keep its trademark or drop it, but cannot modify it.

Note that consumers cannot observe the identities of the agents, but they do observe the histories of the trademarks associated with any agent with whom they might contract. The upfront payment for an agent's product or service is thus contingent on the history of the trademark carried by that agent. We define a *successful trademark* as follows:

DEFINITION 1. A successful trademark is any trademark with a success in the most recent period, and no failure during its recorded history.

This definition limits successful trademarks to those with an S in the prior period. The set of successful trademarks consists of the histories $S, SS, SSS, \ldots$. The maximum two-period working life of agents naturally leads consumers to place great weight on the recent past, since any success recorded for periods prior to the most recent must have been recorded by a different agent than the one currently holding the trademark in question. That is, longer histories must reflect agent discontinuity. The equilibrium we will identify has the property that only successful trademarks as defined above are traded with positive prices.

In the next section, we investigate the effect of trademark sales when the set of agents is fixed. In Section 4, we consider the effect of trademark sales when each agent must incur an entry cost to enter the market, so that the set of agents is endogenously determined.

3. TRADEMARK DILUTION

The amount a consumer is willing to pay for the service offered by a particular agent depends solely on the history of performance recorded by that agent's trademark. Accordingly, we need not distinguish among marks with the same history. Let H_t denote both the set of the available marks and the set of the histories that the marks carry at time t. Since new marks are available for each period, we have $N \in H_t$ for all $t = 1, 2, \ldots$. We seek a perfectly competitive equilibrium with rational expectations, which we refer to below as a price equilibrium.[17] A price equilibrium, with abuse of notation, is an array $\{a_t(\cdot \mid \theta), \mu_t(\cdot), w_t(\cdot), v_t(\cdot)\}$. The first element, $a_t(\cdot \mid \theta)$: $H_t \to \mathrm{H}_t$, records a type θ (both new and continuing) agent's mark selection or purchase outcome at time t, mapping a mark's current history to an available mark at time t.[18] The second element, $\mu_t(\cdot){:}H_t \to \Delta\ (\Theta)$, represents the belief system of consumers at time t, mapping a trademark, $h_t \in H_t$,

[17] For a formal development of a perfectly competitive equilibrium with adverse selection, see, for example, Gale (1992).

[18] When trademark trade is not allowed, agents cannot purchase a different mark, though an agent can always select an N mark by erasing its current mark. When trademark trade is permitted, an agent may keep, sell, erase the current mark, or purchase a different mark. During the mark trading period, we allow for active trading in the sense that an agent can be on both sides of the market; in particular, an agent may first sell its current mark, and then buy back another mark. Note that $a_t(\cdot \mid \theta)$ specifies only the outcome (or allocation) of the trade in trademarks, not the exact actions of an agent during the name trading.

that an agent carries at time t, to a probability measure over the type space of the agents $\Theta = [0, 1]$. The third element, $w_t(\cdot)$: $H_t \to R_+$, represents the equilibrium payments (or prices) for services, mapping any trademark that an agent carries to a nonnegative payment a consumer makes for the service that this agent offers. Finally, $v_t(\cdot)$:$H_t \to R_+$ denotes equilibrium prices for trademarks, mapping any trademark (identified by history h_t) to a nonnegative price for that trademark.

The array $\{a_t(\cdot \mid \theta),\ \mu_t(\cdot),\ w_t(\cdot),\ v_t(\cdot)\}$ constitutes a price equilibrium in our general equilibrium framework if the following conditions hold:

- Given $w_t(\cdot)$ and $v_t(\cdot), a_t(\cdot \mid \theta)$ maximizes the expected payoff of θ-type agents.
- Given $a_t(\cdot \mid \theta)$, the belief system of consumers, $\mu_t(\cdot)$ satisfies the rational expectations condition. That is, it is derived from or consistent with Bayes' rule.
- Given $\mu_t(\cdot), w_t(h_t)$ is the market clearing price for services. In our model, this implies that $w_t(h_t)$ is equal to the expected value of the service provided by an agent with trademark $h_t \in H_t$.
- $v_t(h_t)$ is the market clearing price for any given trademark $h_t \in H_t$.

To simplify equilibrium analysis, we focus on consumers' belief systems with the property that an agent with any history involving at least one F will be regarded as *at best* as good as an agent with no history (a new trademark). Without loss of generality, the payment for an agent with any such history at time t is the same as $w_t(N)$, the payment for an agent with a new trademark.[19] For notational convenience we denote all trademarks with the histories involving at least one F as $\mathcal{F}$. In equilibrium, $w_t(\mathcal{F}) = w_t(N)$. Given this, any agent with an $\mathcal{F}$ mark will be indifferent between holding this mark and discarding this mark (by either erasing the mark or purchasing a mark). We thus focus on the equilibrium in which each agent that possesses an $\mathcal{F}$ mark will choose to discard the mark either by erasing its mark or purchasing a successful mark as a replacement.[20] We begin with the analysis of the base case in which trademark sales are not permitted.

3.1. *Base Case: No Trade in Trademarks.* At time $t = 1$, consumers face two generations of agents. All names are new, N, since the agents that will retire at the end of the period have no prior history. Hence consumers know only that each generation contains agents with types uniformly distributed in the interval [0, 1]. If a consumer randomly chooses an agent, the expected quality of the service

[19] Any payment less than $w_t(N)$ can also be made consistent with our equilibrium.

[20] Suppose there is a "stigma" cost ϵ ($\epsilon > 0$) associated with failed histories. Such a stigma is suggested by Benjamin Franklin's maxim, "Glass, china, and reputation, are easily crack'd, and never well mended" (first appearing in the 1750 edition of *Poor Richard's Almanack*). Then agents will strictly prefer to discard $\mathcal{F}$ marks. Letting $\epsilon \to 0$ we have selected the proposed equilibrium in the limit. Examples abound of firms that have dropped their brands names in the wake of failures. For example, ValuJet Airways dropped its name subsequent to a catastrophic crash in 1996, reappearing as AirTran airlines. WorldCom, battered by accounting scandals, is now MCI. Voicestream, beset with a reputation for poor cellular phone reception, is now T-Mobile. Few new customers will know of these changes. The likelihood of a name change is even greater for owners of less visible trademarks, where the attachment of consumers to a particular mark will be immediately replaced by the stigma of an observed failure.

provided is given by $E\theta = 1/2$. Therefore the payment that consumers are willing to make for an agent in $t = 1$ is $w_1^0(N) = 1/2$, where the superscript 0 denotes the base case, and the subscript 1 denotes time period $t = 1$.

At time $t = 2$, agents that are members of generation 0 retire and are replaced by a new cohort, generation 2. The measure of S marks and the probability of successful service provision are given by

$$m(S) = \int_0^1 \theta d\theta = \frac{1}{2}$$
$$\Pr(S) = E\theta = \frac{1}{2}.$$

In equilibrium agents whose service provision failed erase without cost their history by choosing new trademarks (N). The up-front payments that consumers make for services at the beginning of $t = 2$ are contingent on either S or N agent trademarks.

Given a successful service performance, let $f(\theta \mid S)$ and $E(\theta \mid S)$ be the conditional density function and conditional expected value of θ, respectively. Then

$$f(\theta \mid S) = \frac{f(S \mid \theta) f(\theta)}{\Pr(S)} = \frac{\theta \cdot 1}{1/2} = 2\theta, \text{ and}$$
$$E(\theta \mid S) = \int_0^1 \theta(2\theta)\, d\theta = \frac{2}{3}.$$

Hence $w_2^0(S) = E(\theta \mid S) = 2/3$. Similarly,

$$E(\theta \mid F) = \int_0^1 \theta f(\theta \mid F)\, d\theta = \frac{1}{3}.$$

To determine payments to agents with newly chosen trademarks, imagine that a consumer randomly selects an agent from this pool. With probability 1/3, this agent failed in the previous period, and with probability 2/3, this agent is completely new. When the agent is a member of the new cohort, its expected type is 1/2. When the agent is one that failed in the previous period, its expected type is 1/3. Therefore, the up-front payment to an agent with an N mark is

$$w_2^0(N) = \frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} = \frac{4}{9}.$$

For all periods $t \geq 2$, payments to agents are stationary:

$$w_t^0(S) = \frac{2}{3} \equiv w^0(S), \text{ and}$$
$$w_t^0(N) = \frac{4}{9} \equiv w^0(N).$$

3.2. *Equilibrium with Trademark Sales.* Now consider an alternative setting in which trade in trademarks is permitted. Recall that although consumers are assumed to know the performance associated with a trademark in the past, they cannot observe trademark transfers. First note that permitting the sale of trademarks has no impact at time $t = 1$, since trademarks at that point are nothing more than identifiers for agents. They convey no meaning since no reputations can have been formed. Hence the payment a consumer makes to an agent is as in the base case. Denoting payments in the trademark trade case with a T, we have $w_1^T(N) = w_1^0(N) = 1/2$.

From time $t = 2$ on, the equilibrium analysis becomes less straightforward due to the possibility that trademarks can be traded. When trademarks are identified by histories, the analysis can easily become intractable. To avoid dealing with the complexity of different histories of the trademarks, we will construct a simple form of steady state equilibrium (SSE) in which only successful marks (the marks ended with an S) will be traded with positive prices. Moreover, any successful mark, regardless of its specific history recorded, will be traded at the same price at each period.[21] To save notation, we henceforth write $\mathcal{S}$ to denote any generic successful mark. Our equilibrium construction consists of the following elements (the subscript t is omitted to indicate the stationary equilibrium for $t \geq 2$):

◀$a(\cdot \mid \theta)$▶ Each continuing agent who previously posted an S performance in its first period will continue to hold a successful mark (though the $\mathcal{S}$ mark it keeps after trademark trading need not be the same as the mark owned before name trading); New agents together with continuing agents who failed in their first period will each purchase (and hold) an $\mathcal{S}$ mark with probability 1/3. Agents who do not have $\mathcal{S}$ marks after the trademark trading will start with an N mark. The successful marks are "reshuffled" through the mark trading so that the underlying composition of types of agents is the same for any $\mathcal{S}$ mark, regardless of the number of prior S performances it records.

◀$\mu(\cdot)$▶ For all $\mathcal{S}$ marks, consumers' beliefs about the composition of the mark holders' types are as follows:

- with probability 1/2, the mark holder is a continuing agent that posted an S in its first period;
- with probability 1/3, the mark holder is a new agent; and
- with probability 1/6, the mark holder is an continuing agent that posted an F in its first period.

For all N marks or $\mathcal{F}$ marks, consumers' beliefs concerning the composition of the mark holders' types are as follows:

- with probability 2/3, the mark holder is a new agent, and
- with probability 1/3, the mark holder is an continuing agent who posted an F in its first period.

[21] In Section 5, we provide a set of criteria that support our equilibrium focus.

$\blacktriangleleft w^T(\cdot)\blacktriangleright w^T(\mathcal{S}) = 5/9,\ w^T(N) = w^T(\mathcal{F}) = 4/9.$

$\blacktriangleleft v(\cdot)\blacktriangleright v(\mathcal{S}) = 1/9,\ v(N) = v(\mathcal{F}) = 0.$

PROPOSITION 1. *The array* $\{a(\cdot \mid \theta), \mu(\cdot), w^T(\cdot), v(\cdot)\}$ *described above constitutes a steady-state price equilibrium for* $t \geq 2$.

PROOF. The proof is completed by verifying each of the conditions for the array $\{a(\cdot \mid \theta), \mu(\cdot), w^T(\cdot), v(\cdot)\}$ to constitute an equilibrium.

1. Show that given $\blacktriangleleft w^T(\cdot)\blacktriangleright$ and $\blacktriangleleft v(\cdot)\blacktriangleright$, $\blacktriangleleft a(h_t \mid \theta)\blacktriangleright$ maximizes the expected payoff of a θ-type agent with mark $h_t \in H_t$.

   First we show that a continuing agent that failed in its first period will be indifferent between buying and not buying an $\mathcal{S}$ mark. If such an agent decides not to purchase an $\mathcal{S}$ mark, it will select an N mark. It will be paid at $w^T(N)$ in the current period and if it succeeds in providing a successful service in its last work period (with probability θ), it will have an S mark to sell upon retirement. Hence the expected payoff attached to the decision not to buy an $\mathcal{S}$ mark is given by $w^T(N) + \delta\ \theta\ v(S)$. Similarly, if the agent decides to purchase an $\mathcal{S}$ mark, its expected payoff is given by $w^T(\mathcal{S}) - v(\mathcal{S}) + \delta\theta v(\mathcal{S})$. Since $v(\mathcal{S}) = 1/9 = w^T(\mathcal{S}) - w^T(N)$, the expected payoffs given above are the same, which implies that an continuing agent that failed in its first period is indifferent between buying and not buying an $\mathcal{S}$ mark.

   We now show that a continuing agent who posted an S in its first period is indifferent between selling and retaining its mark. Similarly to the argument above, if it sells its mark, its expected payoff is $v(\mathcal{S}) + w^T(N) + \delta\theta v(S)$. If it retains its mark for its second working period, the expected payoff is $w^T(\mathcal{S}) + \delta\theta v(\mathcal{S})$. Again, given $v(\mathcal{S}) = w^T(\mathcal{S}) - w^T(N)$, the above expected payoffs are the same and hence the agent will be indifferent between selling and keeping an $\mathcal{S}$ mark.

   Finally we show that a new agent will be indifferent between buying and not buying an $\mathcal{S}$ mark. Given the indifference conditions already established above, without loss of generality we assume that when a continuing agent posts an F performance in its first period it will start with a new name, and when a continuing agent posts an S performance in its first period, it will carry it over for its second work period. Given these assumptions, if a new agent starts with a new name, its expected life-time payoff is $w^T(N) + \delta[\theta w^T(S) + (1-\theta)w^T(N)] + \delta^2\theta v(\mathcal{S})$. If the agent buys an $\mathcal{S}$ when it first enters, its expected life-time payoff is $w^T(\mathcal{S}) - v(\mathcal{S}) + \delta[\theta w^T(\mathcal{S}) + (1-\theta)w^T(N)] + \delta^2\theta v(\mathcal{S})$. Again, since $v(\mathcal{S}) = w^T(\mathcal{S}) - w^T(N)$, the two expected payoffs given above are the same. So a new agent is indifferent between buying and not buying an $\mathcal{S}$ name.

   Given these three indifference conditions, $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$ is optimal since no agent will find it profitable to deviate.

2. Show that given $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright, \blacktriangleleft \mu(\cdot)\blacktriangleright$ satisfies the rational expectations condition.

   $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$ implies that for the total measure 1 of $\mathcal{S}$-mark supply (1/2 from the retiring agents and 1/2 from the continuing agents), a measure of 1/2 will be carried over by continuing agents who posted an S in the previous period, a measure of 1/3 will be purchased (and held) by new agents, and a measure of 1/6 will be purchased (and held) by old agents who failed in the previous period.[22] As a result, the belief about the $\mathcal{S}$ mark holders is given by $\blacktriangleleft \mu\ (\cdot)\blacktriangleright$. Similarly, it can be verified that the belief about N mark holders is also given by $\blacktriangleleft \mu\ (\cdot)\blacktriangleright$. According to $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$, no agent will retain an $\mathcal{F}$ mark in equilibrium. Therefore beliefs about $\mathcal{F}$ marks given in $\blacktriangleleft \mu(\cdot)\blacktriangleright$ are trivially consistent with Bayes' rule.
3. Show that given $\blacktriangleleft \mu\ (\cdot)\blacktriangleright, \blacktriangleleft w^T(\cdot)\blacktriangleright$ clears the market for services.
   Given $\blacktriangleleft \mu(\cdot)\blacktriangleright$, we have

$$E(\theta \mid N) = \frac{1}{3} \cdot E(\theta \mid F) + \frac{2}{3} \cdot E(\theta) = \frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} = \frac{4}{9}$$

$$\begin{aligned} E(\theta \mid \mathcal{S}) &= \frac{1}{2} E(\theta \mid S) + \frac{1}{6} E(\theta \mid F) + \frac{1}{3} E(\theta) \\ &= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{9}. \end{aligned}$$

   Therefore, $\blacktriangleleft w^T(\cdot)\blacktriangleright$ clears the market for services as consumers pay the agents up to the expected value of the service performances.
4. Show that $\blacktriangleleft v(\cdot)\blacktriangleright$ are the market clearing prices for trademarks.

   $\blacktriangleleft v(\cdot)\blacktriangleright$, together with $\blacktriangleleft w^T(\cdot)\blacktriangleright$, induces $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$. Given $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$, it can be easily verified that market clears for $\mathcal{S}$ mark.[23] (The markets for N and $\mathcal{F}$ marks are not operated.) By changing $v(\cdot)$, it is apparent that some of the indifference conditions in justifying $\blacktriangleleft a(\cdot \mid \theta)\blacktriangleright$ will fail, which leads to either excessive demand or excessive supply for $\mathcal{S}$ marks. ∎

The key to the above equilibrium construction is that we require the "reshuffling" of $\mathcal{S}$ marks in trademark trading in order to support a simple belief system $\blacktriangleleft \mu(\cdot)\blacktriangleright$, where consumers cannot infer differently given different successful names (characterized by different numbers of past S outcomes). One way to achieve such an outcome in our equilibrium is through two-phase trading: In the first phase, all continuing agents who posted an S performance in the previous period, along with all retiring agents with $\mathcal{S}$ marks, sell their marks to the rest of the active agents, each of whom purchases an $\mathcal{S}$ mark with probability 2/3. In the second phase, with probability 1/2, each new holder of an $\mathcal{S}$ mark sells its mark back to a continuing agent who posted an S in the previous period. As a result of this two-phase trading, each of the continuing agents who posted S performances

[22] These can be verified by using the law of large numbers. Although there are some issues associated with employing the law of large numbers for a continuum of i.i.d. random variables (Judd, 1985), we abuse the law of large numbers here in the manner that is standard in the literature.

[23] We again invoke the law of large numbers.

in their first period will keep an $\mathcal{S}$ mark, and the rest of the active agents will each have an $\mathcal{S}$ mark with probability 1/3. By the law of large numbers, the measure of 1 successful marks is exactly allocated to the measure of 1 agents, and the allocation is completely "reshuffled" in the sense that given any specific history of the $\mathcal{S}$ mark, the underlying composition of the types of agents will be exactly the same. Alternatively, one can imagine assignment of marks through a central clearing house for successful marks. In this setting, all agents (continuing or retiring) with successful marks sell their marks to the clearing house. Each continuing agent who was successful then buys back a successful mark. Finally, each of the rest of the active agents (the continuing agents who failed in the previous period and the new agents) purchases a successful mark with probability 1/3. Each purchased mark is randomly selected from the pool in the clearing house.[24]

Note that for the play at $t = 2$, the reshuffling requirement can be relaxed as the only successful marks are characterized by one period S history. All continuing agents who posted S performances in the previous period may simply keep their marks without trading (whereas the remainder of the active agents, continuing or new, each purchase a retiring S mark with probability 1/3). From $t \geq 2$ on, however, such a simple scheme will not work to support an SSE. The reason is that there will be more than one type of successful mark available in the market. As a result, if continuing agents who posted S performances in the previous period keep their marks, (correct) beliefs about different successful marks, say, S and SS will be different. Consequently the equilibrium payments for those successful marks can differ and the optimality implied in ◀$a(\cdot \mid \theta)$▶ fails. Using the reshuffling apparatus, we are able to overcome the problem and construct the first steady-state competitive equilibrium in this infinite horizon reputation model.

Note that under both the base case and the case permitting trademark trade, the payment to an agent with an N history is the same. We can thus write $w^0(N) = w^T(N) \equiv w(N)$.

The premium received by an agent with a successful mark can be measured by $w^0(S) - w^0(N) = w^0(S) - w(N)$ in the base case and $w^T(\mathcal{S}) - w^T(N) = w^T(\mathcal{S}) - w(N)$ in the case with trademark trade. It can be verified that

$$w^T(\mathcal{S}) - w(N) = \frac{1}{2}(w^0(S) - w(N)) = 1/9. \tag{1}$$

Thus introducing trademark sales results in a direct reduction in the premium commanded by a successful mark, which can be termed as the dilution effect attributable to trademark sales.

3.3. *Agents' Lifetime Payoffs.* We can compare agents' expected lifetime payoffs between the no-trade and trade cases. Let $\Pi_n^0(\theta)$ and $\Pi_n^T(\theta)$ be the expected discounted lifetime payoff for type θ agent of generation $n, n = 0, 1, \ldots$, in the base case and the case with trademark sales, respectively. Then for the agents of first

[24] Note that as is standard for a perfectly competitive equilibrium, in our equilibrium characterization we do not need to specify the exact courses of action leading to the equilibrium outcome.

generation ($n = 0$), who only work for one period in our model, expected lifetime payoffs are given by

$$\Pi_0^0(\theta) = \frac{1}{2} \tag{2}$$

$$\Pi_0^T(\theta) = \frac{1}{2} + \delta\theta v(S). \tag{3}$$

For the second generation (the first two-period cohort, $n = 1$), the expected lifetime payoffs are given by

$$\Pi_1^0(\theta) = \frac{1}{2} + \delta[\theta w^0(S) + (1-\theta)w(N)] \tag{4}$$

$$\Pi_1^T(\theta) = \frac{1}{2} + \delta[\theta w^T(\mathcal{S}) + (1-\theta)w(N)] + \delta^2\theta v(\mathcal{S}). \tag{5}$$

From the third generation on ($n \geq 2$), the expected lifetime payoffs become stationary and are given by

$$\Pi_n^0(\theta) = w(N) + \delta[\theta w^0(S) + (1-\theta)w(N)] \tag{6}$$

$$\Pi_n^T(\theta) = w(N) + \delta[\theta w^T(\mathcal{S}) + (1-\theta)w(N)] + \delta^2\theta v(\mathcal{S}). \tag{7}$$

PROPOSITION 2. *With trade in trademarks, for $\theta > 0$, agents of the first generation are better off, whereas agents of all other generations are worse off.*

PROOF. For the first generation, the result follows from comparing (2) and (3).

For the second generation forward, we can verify that (1) implies

$$\theta w^0(S) + (1-\theta)w(N) = [\theta w^T(\mathcal{S}) + (1-\theta)w(N)] + \theta v(\mathcal{S}). \tag{8}$$

Given (8), the comparison result follows from inspection of Equations (4)–(7). ■

Proposition 2 thus indicates that with trade in trademarks, expected payoffs will be affected for all but the lowest type agents. In particular, all but the lowest type agents from the second generation on will be worse off. The proof of Proposition 2 shows that the dilution effect caused by trademark sales drives the reduction in expected lifetime payoffs for those agents. Trademark sales cause losses to firms that were successful in their first period of services (by 1/9), which are offset by the premium (1/9) that good names command in the future name trade—the net effect is that payments are shifted from the current period to the next period. With time discounting, agents from second generation on are worse off.[25] It can be easily verified that for $n \geq 1, \Pi_n^0(\theta) - \Pi_n^T(\theta) = 1/9 \cdot \delta(1-\delta)\theta$. Thus for any agent from the second generation on, the higher the type, the more the reduction

[25] Note that our comparison result depends on strict discounting assumed throughout this article: With $\delta = 1$, a mere postponement of income will not affect agents' expected lifetime payoffs.

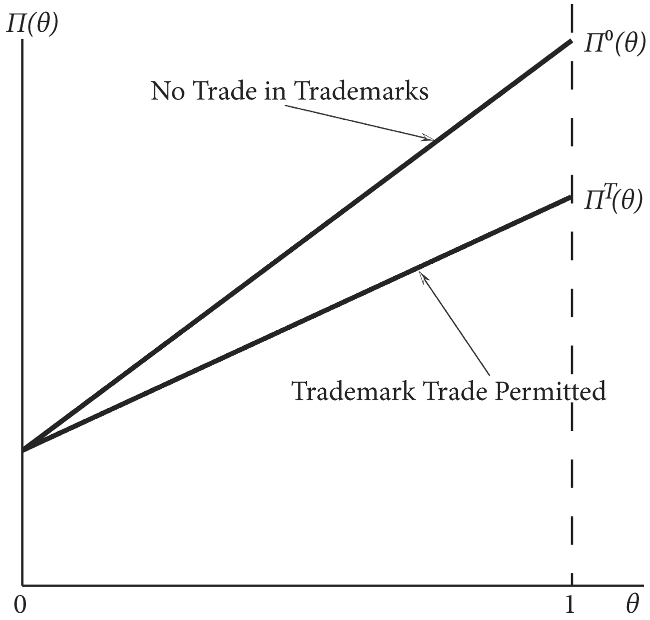

FIGURE 2

STATIONARY LIFETIME EXPECTED PAYOFFS

in expected payoffs. The comparison of the stationary expected lifetime payoffs (for $n \geq 2$) is illustrated in Figure 2.

It is worth noting that although expected payoffs are affected for all but the lowest type agents, welfare is not affected by introducing trademark sales. To see this, since the expected consumer surplus is zero in our model, we only need to verify that the total expected payoff of all the agents is not affected. Indeed it can be verified that when discounting back to time $t = 1$, for any given agent type θ, the total expected lifetime payoffs summing over all agents of all generations are the same under both market structures—with trademark trade the gain of the first generation exactly offsets the loss of all the rest generations. Another way to see this is to compare the expected surplus contributed by each agent. Since the expected surplus contributed by each type θ agent is the same (which is θ), and the set of agents are also the same (which is $[0, 1]$), the total expected surplus remains the same.

The above welfare implication obviously hinges on the assumption that the set of agents is fixed and exogenously given. It should be expected that, if this assumption is relaxed, that is, if the set of agents is endogenously determined, then it may not be the case that the welfare will remain unaffected. One way to model endogenous entry is to assume that there is an entry cost for each agent operating in the service market. Given agent entry, and in light of Proposition 2, we should anticipate that though the entry of the first generation agents (who live for one period only) will not be affected, the entry of the agents of all the other generations will be affected. In the next section, we investigate the welfare implication of such effects.

4. TRADEMARK SALES WITH AGENT ENTRY

In this section, we introduce an up-front cost that agents must incur in order to enter the market for the service in question. This entry cost can include the cost of training and/or an opportunity cost of devoting oneself to providing the service. This cost, once incurred, need not affect either the ability or the cost of service provision for agents that have chosen to enter. Thus a medical doctor or a lawyer may have to invest in significant amounts of training in preparation for entering his or her chosen profession. Once the training has been acquired, however, a more able physician need not incur any greater cost in developing a diagnosis or performing a surgical procedure than a lower type, and similarly the costs incurred by a lawyer in representing a client in a dispute need not depend on the ability of the lawyer. Moreover, the training, even if observable by consumers, need not signal agent quality. Even a fine school is likely from time to time to produce graduates in which it can take no pride. In this section, we consider explicitly the entry decision of agents that must incur an avoidable fixed cost as a condition of entry. We continue to assume an adverse selection setting for agents that have chosen to enter. Their costs of delivering the service they compete to offer, including costs of effort, are independent of their exogenous quality. We assume that all agents make entry decisions independently and simultaneously. For ease of analysis we will focus on the stationary state (starting from $t = 3$).[26]

We continue to assume that the measure of potential agents is fixed and the type of potential agents is denoted by $\theta \sim U[0, 1]$. We will first characterize the equilibrium with entry and then examine its welfare implications.

4.1. *Equilibrium with Entry.*

It is easily verified that when entry cost is lower than $\frac{4}{9}(1 + \delta)$, entry will be unaffected by the trademark trading rule. When entry cost is higher than $1 + \delta$, no agent can enter the market with positive profit. Letting $\underline{c} = \frac{4}{9}(1 + \delta)$ and $\bar{c} = 1 + \delta$, we thus focus on the case with $c \in (\underline{c}, \bar{c})$. We continue to focus on the equilibrium in which agents with history F will erase the stigma associated with their prior failure by choosing new trademarks, and we will again focus on the "reshuffling" equilibrium in the spirit characterized by Proposition 1. That is, only $\mathcal{S}$ marks will be traded at positive prices, and the payments to successful marks are the same in each period regardless of the full history associated with a specific mark.

PROPOSITION 3. *Given $c \in (\underline{c}, \bar{c})$, there exists a unique equilibrium characterized by a (unique) entry threshold $\theta^{*k} \in (0, 1), k \in \{0, T\}$, such that all agents with types*

[26] Introducing entry in the nonstationary state becomes cumbersome without yielding much additional insight. Our focus on the stationary state is equivalent to considering the infinite horizon model running from time $-\infty$ to time $+\infty$. Such simplification is justifiable, as our stationary equilibrium remains to be an equilibrium in the model with such extended infinite horizon.

$\theta \in [\theta^{*k}, 1]$ *enter the market whereas all agents with types* $\theta \in [0, \theta^{*k})$ *choose not to enter. Moreover*, $\theta^{*0} < \theta^{*T}$.

PROOF. See the Appendix.

The proof is straightforward. We first show that any equilibrium with entry must have the threshold property. We then show that such a threshold equilibrium does exist and is unique.

For $c \in (\underline{c}, \bar{c})$, we show that $\theta^{*T} > \theta^{*0}$. Hence the market without trademark sales admits more agents. This result is intuitive. As allowing trademark sales decreases agents' lifetime payoffs, the measure of agents for whom expected payoffs surpass opportunity costs falls. Even though the burden falls most heavily on high-type agents, allowing trade has its effect on entry at the margin, and the agents that such trade discourages are the lowest quality that the market would otherwise admit.

It is straightforward to verify that $\partial w(N, \theta^*)/\partial\, \theta^* > 0$ for $\theta^* \in (0, 1)$. Given $\theta^{*T} > \theta^{*0}$, the intercept of the expected lifetime payoff schedule (gross of entry cost) is higher in the market with trademark sales, which implies that the lifetime expected payoff functions are as illustrated in Figure 3.

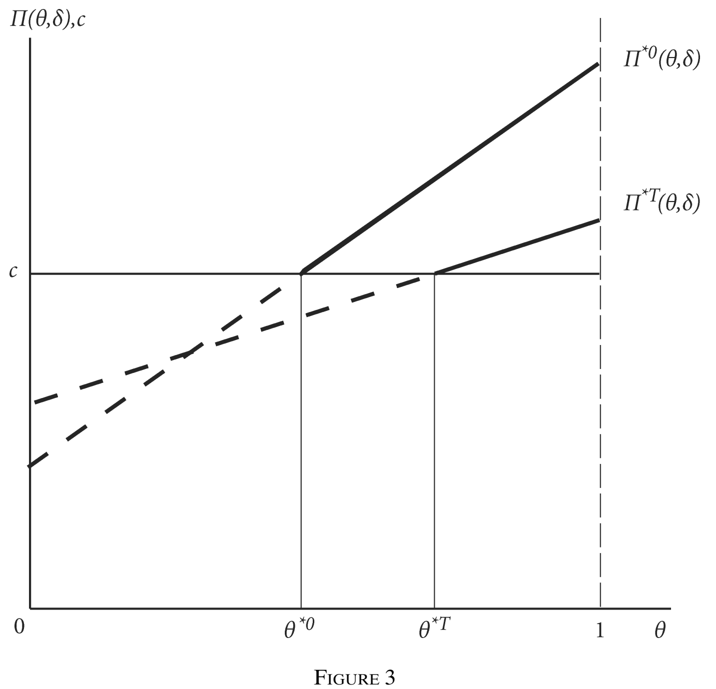

FIGURE 3

EQUILIBRIUM LIFETIME EXPECTED PAYOFFS WITH ENTRY

4.2. *Welfare Implications.* To determine the welfare effects of permitting trade in trademarks, consider first the socially optimal level of entry. Given any specific period, say t_0, in the steady state, we assume that the social planner's objective is to determine the set of agents to maximize the expected total surplus starting from time t_0. Given that the expected surplus in each period remains the same in the steady state, the social planner's problem is equivalent to maximizing each period's expected surplus by selecting an entry threshold θ^*. In each period, agents who are in the market consist of two generations, the continuing and the new. For each type (θ), a continuing agent generates expected surplus θ, and a new agent generates expected surplus $\theta - c$ (as each new agent incurs an entry cost c). The social planner thus chooses an entry threshold (θ') to maximize the following objective function:

$$\int_{\theta'}^{1} \theta \, d\theta + \int_{\theta'}^{1} (\theta - c) \, d\theta.$$

The unique socially optimal entry threshold $\theta^* = c/2$, which is intuitive as the average entry cost for both the new and continuing generation agents in each period is $c/2$.

The following proposition summarizes the welfare ranking between two rules:

PROPOSITION 4. *There exists a unique entry cost level* $c^* \in (\underline{c}, \bar{c})$ *such that*

(i) with entry cost $c \leq \underline{c}$, *or* $c \geq \bar{c}$, *either all potential agents enter the market or no agent can afford the entry cost to enter the market irrespective of the rule governing trademark sales, so permitting such sales has no welfare effect*;
(ii) with $c \in (\underline{c}, c^*)$, *the market with trademark trade welfare dominates that without trademark trade*;
(iii) with $c \in (c^*, \bar{c})$, *the market where trademark trade is prohibited welfare dominates that with trademark trade.*

PROOF. See the Appendix.

Proposition 4 suggests a clear parameterization for a welfare comparison between the market with trademark sales and the benchmark market. To understand the intuition, for any given entry level $c \in (\underline{c}, \bar{c})$ we start with the socially optimal entry benchmark where the lowest type to enter the market is $c/2$. Let $\Pi^k(\theta, \theta^*)$ denote the expected lifetime payoff for a type θ agent given the entry threshold θ^*. Then for the socially optimal "marginal type," $\theta = c/2$, the expected gain from entry is given as follows:

$$\Pi^k\left(\frac{c}{2}, \frac{c}{2}\right) - c \equiv \Psi^k\left(\frac{c}{2}\right).$$

As shown in the proof of Proposition 3, $\Pi^k(\frac{c}{2}, \frac{c}{2})$ is increasing in c (as a higher entry threshold pushes up the average quality of the agents in the market pool, the expected lifetime payoff for the marginal entrant also increases). But this term increases at a rate less than 1, so $\Psi^k(\frac{c}{2})$, the expected gain from entry for the socially optimal marginal entrant, is (strictly) decreasing in c. This implies that there is a unique c^k such that $\Psi^k(\frac{c^k}{2}) = 0$ (and it is easily verified that $c^T < c^0$ due to the trademark dilution effect). The implication is that the socially optimal marginal entrant is indifferent between entering the market and remaining out only when the entry cost is c^k. In other words, the socially optimal entry coincides with equilibrium entry only when $c = c^k$ (in each market structure k).

If $c \neq c^k$, equilibrium entry will be either excessive or insufficient. First consider the case where $c < c^T < c^0$. In this case, $\Psi^k(\frac{c}{2}) > 0$. The socially optimal marginal entrant strictly prefers to enter the market, which implies that the equilibrium threshold $\theta^{*k} < c/2$. Since the pool of types $[\theta^{*k}, c/2]$ enter the market contributing negative surplus (net of per period average entry cost), equilibrium entry is excessive. As a result, allowing trademark sales improves welfare (by making entry more difficult).

Next consider the case where $c^T < c^0 < c$. In this case, $\Psi^k(\frac{c}{2}) < 0$. The socially optimal marginal entrant strictly prefers not to enter, which implies that the equilibrium threshold $\theta^{*k} > c/2$. Since the pool of types $[c/2, \theta^{*k}]$ who choose not to enter would have contributed positive surplus (net of per period average entry cost), equilibrium entry is insufficient. In this case banning trademark sales improves welfare by making entry more attractive.

For the intermediate case, $c \in (c^T, c^0)$, we have $\Psi^T(\frac{c}{2}) < 0 < \Psi^0(\frac{c}{2})$. Following the same arguments as above, we have $c^T < \theta^{*0} < c/2 < \theta^{*T} < c^0$. Entry is excessive without trademark sales and insufficient with trademark sales. In this case we show that there exists a unique cutoff $c^* \in (c^T, c^0)$. When $c \in (c^T, c^*)$ the equilibrium θ^{*T} brings the entry closer to the social optimal compared to θ^{*0}; thus allowing trademark sales is preferable. When $c \in (c^*, c^0)$, the situation is reversed and banning trademark sales is preferable.

Our welfare result has the following interpretation. Due to information asymmetry, consumers cannot distinguish relatively low-quality agents from better rivals. Our assumption that any agent with a failed history can erase its past at no cost to itself by posing as a new agent makes it even easier for low-quality agents to enter the market. Our benchmark model thus implies that entry will be excessive when entry cost is low. The appropriate response to such excessive entry is to reduce the market returns to those low-type agents. This can be accomplished by permitting trademark sales. Although allowing trademark sales also dilutes the payoffs to high-type agents, they are not marginal entrants, and hence their entry will be unaffected, suggesting no harm to welfare. However, when entry cost is high, our concern shifts to insufficient entry by better agents. Since allowing trademark sales reduces payoffs particularly to better agents (recall that the higher the types, the more striking the dilution effect), banning trademark sales to eliminate dilution becomes welfare improving.

It is worth emphasizing at this point how surprising our results are. If the welfare problem is one of high entry costs that cause too few agents to enter the market, one might expect that the problem is best handled by allowing successful agents to sell their marks. This intuition is wrong. Our analysis shows that the appropriate solution is to generate more high-type entrants by protecting them from the adverse effects of dilution of their marks. This can be accomplished by preventing the marks from falling into the hands of lower quality agents through trademark sales. Our result thus suggest a justification for the U.S. trademark law of banning trademark trade "in gross," that is, banning trademark trade can prevent payoffs of high-type agents from being diluted.

Finally, note that our welfare result summarized in Proposition 4 hinges on strict discounting. Without discounting (i.e., if $\delta = 1$), trademark dilution is absent and agents are indifferent between the two market structures. Consequently, allowing trademark trade has no effect on entry. It should be noted, however, that our welfare results hold for any discount factor less than unity.

5. DISCUSSION

Our analysis hinges crucially on the equilibrium we characterize. As the problem of multiple equilibria is endemic in the dynamic game literature, our equilibrium is not the only available candidate. We could, for example, trivially construct many other equilibria by simply perturbing beliefs about an $\mathcal{F}$ mark. But such equilibria are essentially the same as the one in our analysis, since the payments or prices for successful marks are basically the same on the equilibrium path. Are there other equilibria that differ substantially from ours? How restrictive is the construction of our equilibrium? Instead of identifying all other equilibria (a daunting task) or looking for equilibrium refinements (not a fruitful pursuit in reputation models with pure adverse selection), we suggest the following criteria in support of our equilibrium.

We claim that any sensible equilibrium for analyzing trademark sales should possess the following three features:

$\mathcal{A}$: **Adverse belief about an $\mathcal{F}$ mark**. If a trademark's history contains at least one F, then consumers believe (or infer) that the trademark holder's type is *at best* as good as that of a new trademark holder.

$\mathcal{M}$: **Monotonicity of $\mathcal{S}$ mark values**. The prices of successful trademarks, and the payments to agents with successful trademarks in each period $t, t \geq 2$, satisfy the following monotonicity requirements:

$$w_t(S) \leq w_t(SS) \leq w_t(SSS) \leq \cdots$$
$$v_t(S) \leq v_t(SS) \leq v_t(SSS) \leq \cdots$$

$\mathcal{N}$: **No sorting of reputation**. No successful trademark can serve as sorting device that separates higher-type agents from lower-type agents.

Condition ($\mathcal{A}$) implies that in equilibrium, agents will present consumers with either new or successful trademarks. Thus only successful marks will be traded at positive prices. This condition is not innocuous, but without this restriction, the equilibrium analysis would easily become intractable as different marks would be traded at different prices. Condition ($\mathcal{M}$) is a natural inference from the often observed advertising practice for revealing firms' longevity. It is implausible that firms would find the value of their trademark lowered by the addition of another successful outcome to their performance history. Were this property to fail, at least some successful agents would wish to erase some part of their history, thereby attempting to pose as less successful. Thus we may also interpret this property as a "free disposal" assumption for trademark histories. Condition ($\mathcal{N}$) reflects a major result obtained by Tadelis (1999, 2002, 2003), namely that no sorting can occur in the OLG reputation model.[27] This no-sorting condition is also consistent with the main result in Mailath and Samuelson (2001), who show that in equilibrium, very strong reputations are more likely to be purchased by bad types whereas average reputations are more likely to be purchased by good types.

If each of these three conditions is satisfied, only successful marks will be traded at positive prices and all successful marks will be valued identically in each period regardless of their exact histories (a formal proof is provided in the Appendix). Note that these two properties are exactly the focus or the starting point in our equilibrium construction.

Conditions ($\mathcal{A}$), ($\mathcal{M}$), and ($\mathcal{N}$) certainly impose some restrictions on the equilibrium we consider. But we believe that an equilibrium satisfying all these conditions captures the most important features of trademark sales and thereby provides a useful platform on which to conduct our analysis.[28] Since all successful marks are priced identically, one clear implication is that consumers cannot infer differently given different successful marks. The reshuffling apparatus employed in our equilibrium construction exactly supports a sensible equilibrium that satisfies all the conditions identified above. Without this apparatus, the existence of an equilibrium satisfying these conditions is an open question.

6. SUMMARY AND CONCLUSIONS

We have interpreted dilution as an external effect of the actions of another agent that causes consumers to lower their expectations for the service provision that the holder of a successful trademark will provide. Trademark trade permits trademarks to change hands out of the view of consumers. The ability of trademarks to denote promising traders is thereby impaired. Trademark trade may be beneficial when trademark owners are induced to provide effort in order to increase the probability of having successful marks to sell, but in our adverse selection setting, such trade only facilitates dilution and thereby makes agents worse off in the steady state. Note also that in our model, the damage caused by the actions of

[27] For two type agents case, Tadelis (1999, 2002) shows that the no-sorting property holds when $w(SF) > w(N)$, and for continuous type agents case, Tadelis (2003) shows that the no-sorting holds when $w(SS) = w(S) = w(SF)$ (a consequence from the "random matching" condition).

[28] One direct implication from condition ($\mathcal{A}$) is that firms have neither the luxury nor the incentive to rebuild trademarks subsequent to an unfavorable outcome. Note, however, that in practice, a firm need not drop its mark whenever it fails in some portion of its activities. Instead, we simply require that consumers observe whether or not a firm has been successful at the end of a period. That success is the result of the firm's performance during the entire period, performance that is a composite of performance experiences over the period. A successful law firm need not win all of its cases to be considered a success, but one that fails consistently, and hence ultimately, should be expected to reorganize and reemerge under a new banner. All that we require is that consumers agree in their assessments of what constitutes success and failure.

a particular agent is vanishingly small (due to the zero measure of an individual type). Thus the requirement that a trademark holder show that the actions of a particular rival causes measurable damage is an impossible task in our model.[29]

More specifically, in an infinite horizon OLG reputation model, we have compared the United States (no trade) rule with the more common rule permitting trademark sales. In our equilibrium, allowing trademark sales affects all but the lowest type agents. For higher types, all but the first generation of agents are worse off when marks can be traded. For those agents, we demonstrate that, due to trademark dilution effect, the benefit of allowing trademark sales is exactly offset by the income reduction during their working periods. With time discounting, they are strictly worse off. Better agents incur larger penalties from dilution of the value of their trademarks. Thus when endogenous entry is taken into account, allowing trade in trademark affects the entry of all but the first generation agents. By focusing on the stationary state analysis, our welfare implications are nicely parameterized by the entry cost level. When entry cost is low, our model typically implies excessive entry. In this case, allowing trademark trade is preferable as the dilution arising from trademark sales reduces expected payoffs and makes entry more difficult for the lower-type agents at the margin of the entry decision. When entry cost is high, entry will be insufficient and banning trademark trade is preferable, as trademark sales would otherwise dilute the ability of higher-type agents to enter the market.[30]

Since our welfare implication is not uniform, our results do not provide direct support for either the GATT/TRIPs rule permitting trademark trade or the U.S. rule banning trademark sales. Instead, our contribution is to suggest a reputation model to assess these two opposing rules. In our model, dilution has no effect on consumers, whose rational expectations ensure that they pay the expected value for trademarked services, so that only trademark holders are affected. Our surprising result is that counter to intuition that trademark holders benefit from the ability to sell their marks, the effect we find is negative for all but the first generation agents. The ability to sell a trademark provides a clear benefit to a retiring agent with a successful trademark in hand, but that benefit is more than offset in expectation for those agents who spend their working years receiving lowered compensation due to the dilution that trademark trade facilitates.

Our results depend on our assumption that performance differences among agents arise from differences in ability. To the extent that agents can alter outcomes by exerting effort (Tadelis, 2002), denying agents the ability to profit from sales of trademarks built on effort can discourage such effort. Our results indicate that effort promotion needs to be balanced against the dilution of the value of a good trademark that trademark trade facilitates. Note also that the U.S. rule permits the sale of trademarks when that sale is part of a transaction conveying an underlying business. Broader transactions of this form are more likely to convey to the new owner incentives for effort together with the ability to deliver services in accordance with the trademark's reputation.

[29] The Supreme Court has imposed such a requirement. See Moseley v. V Secret Catalogue, 537 U.S. 418 (2003).

[30] It is worth noting that although dilution is always present with trademark trade, such dilution can be desirable when it discourages entry by low-quality agents.

From a broader perspective, our results suggest that "propertization" of trademarks should be partial. Although defining property rights for trademark owners is an important legal function, and gives successful trademark owners the ability to earn a premium for their successes, permitting such rights to be traded can be counterproductive for these agents, and for welfare as well under some plausible circumstances.

APPENDIX

PROOF OF PROPOSITION 3. First we argue that any entry equilibrium is a threshold equilibrium. To see this, consider any two given types $\theta^{'}$ and $\theta^{''}$, where $\theta^{'} < \theta^{''}$. Suppose in equilibrium type $\theta^{'}$ is in the market, then we claim that type $\theta^{''}$ must also be in the market (with positive expected lifetime payoff). This is due to the following reasons. First, since a single type has measure zero, the addition of type $\theta^{''}$ would not affect the composition of the agent types in the market. Hence the equilibrium payments or prices for services remain the same. Second, given the same prices for services, the expected lifetime payoff for a type $\theta^{''}$ agent will be higher than that for a type $\theta^{'}$ agent, since a higher type results in a higher probability of sucesses. Thus if type $\theta^{'}$ can enter the market with positive expected lifetime payoff, type $\theta^{''}$ can afford the entry as well, which implies that any equilibrium with entry must be characterized by a minimal type that enters the market (the entry threshold).

Next we show that such a threshold equilibrium exists and is unique. More specifically, we need to show that there is a unique threshold $\theta^{*k} \in (0, 1)$ such that the following two conditions hold:

(1) given that all types $[\theta^{*k}, 1]$ enter the market, a type $\hat{\theta} \in [0, \theta^{*k})$, $k \in \{0, T\}$, cannot enter the market profitably, and
(2) given that all types $[\theta^{*k}, 1]$ except type $\hat{\theta} \geq \theta^{*k}$, $k \in \{0, T\}$, enter the market, type $\hat{\theta}$ cannot be better off by remaining out of the market.

We first determine stationary state market payments for agents, given that agents with types $[\theta^{*k}, 1]$, $k \in \{0, T\}$, participate in the market.

Case 1: No trademark sales. The probability of a success for a service in each period is given by

$$\Pr(S) = E(\theta \mid \theta \geq \theta^*) = \int_{\theta^*}^{1} \theta \, d\theta / (1 - \theta^*) = (1 + \theta^*)/2$$

Given S or F in the previous period, the conditional density functions can be computed as follows:

$$f(\theta \mid S) = \frac{f(S \mid \theta) f(\theta)}{\Pr(S)} = \frac{\theta \cdot 1/(1 - \theta^*)}{\frac{1}{2}(1 + \theta^*)} = \frac{2\theta}{1 - \theta^{*2}}, \tag{A.1}$$

and

(A.2) $$f(\theta \mid F) = \frac{f(F \mid \theta) f(\theta)}{\Pr(F)} = \frac{(1-\theta)\cdot 1/(1-\theta^*)}{(1-\theta^*)/2} = \frac{2(1-\theta)}{(1-\theta^*)^2}.$$

From (A.1) and (A.2) we have

$$E(\theta \mid S) = \int_{\theta^*}^{1} \frac{2\theta^2}{1-\theta^{*2}}\, d\theta = \frac{2}{3}\left[\frac{1+\theta^* + \theta^{*2}}{1+\theta^*}\right]$$

and

$$E(\theta \mid F) = \int_{\theta^*}^{1} \theta \frac{2(1-\theta)}{(1-\theta^*)^2}\, d\theta = \frac{2}{(1-\theta^*)^2}\left[\frac{1}{6} - \frac{1}{2}\theta^{*2} + \frac{1}{3}\theta^{*3}\right].$$

Hence, the payment for an agent with trademark S is

(A.3) $$w^0(S, \theta^*) = \frac{2}{3}\left[\frac{1+\theta^* + \theta^{*2}}{1+\theta^*}\right].$$

To determine the payment for an agent with a new trademark, $w^0(N, \theta^*)$, we continue to focus on the equilibrium in which F marks are replaced by new marks. The total measure of this category of agents is given by

$$m(F) = \int_{\theta^*}^{1} (1-\theta)\, d\theta = \frac{1}{2}(1-\theta^*)^2.$$

After agents with F histories choose new trademarks, the total measure of agents with N histories will be $m(N) = (1-\theta^*) + \frac{1}{2}(1-\theta^*)^2 = (1-\theta^*)(\frac{3}{2} - \frac{\theta^*}{2})$. Therefore the payment for agents with N trademarks can be computed as follows:

(A.4) $$\begin{aligned} w^0(N, \theta^*) = E(\theta \mid N) &= \frac{(1-\theta^*)^2/2}{(1-\theta^*)\left(\frac{3}{2} - \frac{\theta^*}{2}\right)} E(\theta \mid F) \\ &\quad + \frac{1-\theta^*}{(1-\theta^*)\left(\frac{3}{2} - \frac{\theta^*}{2}\right)} E(\theta \mid \theta \geq \theta^*) \\ &= \frac{1}{(3-\theta^*)(1-\theta^*)}\left[\frac{4}{3} - 2\theta^{*2} + \frac{2}{3}\theta^{*3}\right]. \end{aligned}$$

Case 2: Market with trademark sales permitted. In this case, we will again focus on the "reshuffling" equilibrium in the spirit characterized by Proposition 1. That is, only $\mathcal{S}$ marks will be traded at positive prices, and the payments to successful marks are the same in each period regardless of the full history associated with a specific mark. At the end of each period, the $\mathcal{S}$ marks will be generated by both the retiring agents and the continuing agents. The total measure of $\mathcal{S}$ trademarks is given by

$$m(\mathcal{S}) = 2\int_{\theta^*}^{1} \theta\, d\theta = 1 - \theta^{*2} \tag{A.5}$$

so that the total supply of $\mathcal{S}$ trademarks in each period has measure $(1-\theta^{*2})$. Again we focus on the equilibrium in which (1) one half of the $\mathcal{S}$ marks (with measure $(1-\theta^{*2})/2$, after being "reshuffled") are kept by the continuing agents who posted S performances in the previous period, and (2) the rest of the $\mathcal{S}$ marks (measure $(1-\theta^{*2})/2)$) are uniformly "rationed" to the rest of the agents.[31] Taking this mark reshuffling into account we can compute the payments to agents in the stationary state as follows:

$$\begin{aligned} w^T(N,\theta^*) &= E(\theta \mid N) = w^0(N,\theta^*) \equiv w(N,\theta^*) \\ w^T(\mathcal{S},\theta^*) &= E(\theta \mid \mathcal{S}) \\ &= \frac{1}{2}E(\theta \mid S) + \frac{1}{2}w(N,\theta^*) \\ &= \frac{1}{2}\cdot\frac{2}{3}\frac{1+\theta^*+\theta^{*2}}{1+\theta^*} + \frac{1}{2}w(N,\theta^*) \\ &= \frac{1}{3}\left[\frac{1+\theta^*+\theta^{*2}}{1+\theta^*}\right] + \frac{1}{2}w(N,\theta^*). \end{aligned} \tag{A.6}$$

The price for an $\mathcal{S}$ trademark is given by

$$v(\mathcal{S},\theta^*) = w^T(\mathcal{S},\theta^*) - w(N,\theta^*).$$

Given that the set of agents with types $[0,\theta^*]\setminus\{\hat{\theta}\}$ are already in the market, let $\Pi^k(\hat{\theta},\theta^*)$, $k\in\{0,T\}$, denote the expected lifetime payoff for a type-$\hat{\theta}$ agent if it enters the market. Since one particular type has measure zero and does not affect the equilibrium payments or prices, we have

$$\Pi^0(\hat{\theta},\theta^*) = w(N,\theta^*) + \delta[\hat{\theta}w^0(\mathcal{S},\theta^*) + (1-\hat{\theta})w(N,\theta^*)], \quad \text{and} \tag{A.7}$$

$$\begin{aligned} \Pi^T(\hat{\theta},\theta^*) = {} & w(N,\theta^*) + \delta[\hat{\theta}w^T(\mathcal{S},\theta^*) + (1-\hat{\theta})w(N,\theta^*)] \\ & + \delta^2\hat{\theta}v(\mathcal{S},\theta^*). \end{aligned} \tag{A.8}$$

It is easily verified that $\hat{\theta}w^0(\mathcal{S},\theta^*) + (1-\hat{\theta})w(N,\theta^*) = \hat{\theta}w^T(\mathcal{S},\theta^*) + (1-\hat{\theta})w(N,\theta^*) + \hat{\theta}v(\mathcal{S},\theta^*)$. Therefore $\Pi^0(\hat{\theta},\theta^*) > \Pi^T(\hat{\theta},\theta^*)$ for all $\hat{\theta} > 0$. So given essentially the same set of entrant agents (which can be different for types with

[31] The new agents, and the continuing agents who posted F performances, will each obtain an $\mathcal{S}$ mark with probability $(1+\theta^*)/(3-\theta^*)$.

measure zero), essentially all agents are strictly worse off in market with trademark sales.

It is also easily verified that $w^0(S, \theta^*) - w(N, \theta^*) = 2(w^T(\mathcal{S}, \theta^*) - w(N, \theta^*))$. Based on this we can re-write (A.7) and (A.8) as follows:

$$\Pi^0(\hat{\theta}, \theta^*) = (1+\delta)w(N, \theta^*) + 2\delta(w^T(\mathcal{S}, \theta^*) - w(N, \theta^*))\hat{\theta} \tag{A.9}$$

$$\Pi^T(\hat{\theta}, \theta^*) = (1+\delta)w(N, \theta^*) + \delta(1+\delta)(w^T(\mathcal{S}, \theta^*) - w(N, \theta^*))\hat{\theta}. \tag{A.10}$$

The expected payoff (gross of entry cost) for the marginal type agent is given by $\Pi^k(\theta^*, \theta^*), k \in \{0, T\}$. Differentiating with respect to θ^*, we have

(A.11)

$$\frac{d\Pi^0(\theta^*, \theta^*)}{d\theta^*} = \frac{2(\theta^{*4} - 4\theta^{*3} - 3\theta^{*2} + 3\delta\theta^{*2} - 2\delta\theta^* + 10\theta^* + 8 + 11\delta)}{3(1+\theta^*)^2(\theta^* - 3)^2} > 0$$

$$\frac{d\Pi^T(\theta^*, \theta^*)}{d\theta^*} = \frac{1+\delta}{3(1+\theta^*)^2(\theta^* - 3)^2}$$

$$\cdot [(2-\delta)\theta^{*4} - (8-4\delta)\theta^{*3} - 6(1-\delta)\theta^{*2} + (20-12\delta)\theta^* + 16 + 3\delta] > 0.$$

Thus $\Pi^k(\theta^*, \theta^*)$ is strictly increasing in $\theta^*, k \in \{0, T\}$. It is easily verified that $\Pi^k(0,0) = \frac{4}{9}(1+\delta)$, and by L'Hopital's rule $\Pi^k(1,1) = 1+\delta, k = 0, T$. Therefore when $c \in (\frac{4}{9}(1+\delta), 1+\delta)$, by the continuity of Π^0 and Π^T, there exists a unique $\theta^{*k} \in (0,1), k \in \{0, T\}$ such that

$$\Pi^k(\theta^{*k}, \theta^{*k}) = c. \tag{A.12}$$

By (A.9) and (A.10), $\Pi^k(\hat{\theta}, \theta^{*k}) - c$ is strictly increasing in $\hat{\theta}$. Thus θ^{*k} characterizes the unique equilibrium satisfying the two conditions specified at the beginning of this proof.

Moreover, that $\frac{4}{9}(1+\delta) < c < 1+\delta$ implies $\theta^{*T} \in (0,1)$. Since $\Pi^0(\theta, \theta^*) > \Pi^T(\theta, \theta^*)$ for $\theta > 0$, we have

$$\Pi^0(\theta^{*T}, \theta^{*T}) > \Pi^T(\theta^{*T}, \theta^{*T}) = c.$$

The strict monotonicity of $\Pi^0(\theta^*, \theta^*)$ thus implies that $\theta^{*T} > \theta^{*0}$. ■

PROOF OF PROPOSITION 4. The cases when $c \leq \underline{c} = \frac{4}{9}(1+\delta)$ (all agents enter) and $c \geq \bar{c} = 1+\delta$ (high entry cost precludes all but measure 0 agents from entering) are trivial. We thus focus on the case when incomplete entry is possible, $c \in (\underline{c}, \bar{c})$.

Define $\Psi^k(x) = \Pi^k(x, x) - 2x,\ x \in (\frac{\underline{c}}{2}, \frac{\bar{c}}{2})$. It can be easily verified that $\Psi^k(\frac{\underline{c}}{2}) > 0$ and $\Psi(\frac{\bar{c}}{2}) < 0$. Based on (A.11) it can also be verified that $\frac{d\Psi^k(x)}{dx} < 0,\ k \in \{0, T\}$. Therefore there exists a unique pair (c^0, c^T) such that

$$\Pi^0\left(\frac{c^0}{2}, \frac{c^0}{2}\right) - c^0 = 0, \quad \Pi^T\left(\frac{c^T}{2}, \frac{c^T}{2}\right) - c^T = 0.$$

The interpretation of c^k is that when $c = c^k$, the equilibrium entry induced by market k, $k \in \{0, T\}$, is socially optimal when the corresponding trademark trading rule is applied. Moreover, $\Pi^0(x, x) > \Pi^T(x, x)$ for $x \in (\frac{\underline{c}}{2}, \frac{\bar{c}}{2})$ implies that $\Psi^0(x) > \Psi^T(x)$. Therefore $\Psi^0(\frac{c^T}{2}) > \Psi^T(\frac{c^T}{2}) = 0$. Since $\Psi^0(\frac{\bar{c}}{2}) < 0$, by continuity of $\Psi^0, c^0 > c^T$.

If $c > c^0$, $\Psi^0(\frac{c}{2}) < \Psi^0(\frac{c^0}{2}) = 0$. Thus $\Pi^0(\frac{c}{2}, \frac{c}{2}) < c = \Pi^0(\theta^{*0}, \theta^{*0})$ and hence $\theta^{*T} > \theta^{*0} > \frac{c}{2}$, which implies that entry is insufficient under both market structures and banning trademark sales improves welfare.

If $c < c^T$, $\Psi^T(\frac{c}{2}) > \Psi^T(\frac{c^T}{2}) = 0$. Thus $\Pi^T(\frac{c}{2}, \frac{c}{2}) > c = \Pi^T(\theta^{*T}, \theta^{*T})$ and hence $\frac{c}{2} > \theta^{*T} > \theta^{*0}$, which implies that entry is excessive under both market structures and allowing trademark sales improves welfare.

If $c \in (c^T, c^0)$, we have $\theta^{*0} < \frac{c}{2} < \theta^{*T}$. That is, entry is excessive in market 0, and insufficient in market T. Intuitively the welfare ranking will depend on which market is "closer" to the social optimum. We claim that there exists a unique $c^* \in (c^T, c^0)$ such that when $c \in (c^T, c^*)$, market T welfare dominates market 0; when $c \in (c^*, c^0)$, market 0 welfare dominates market T, and two markets generate the same welfare when $c = c^*$. The proof is completed in the following two steps.

Step 1: Market 0 welfare dominates market T if and only if $\frac{c}{2} - \theta^{*0} < \theta^{*T} - \frac{c}{2}$.

Under market 0, there will be excessive entry in equilibrium. The welfare loss in each period is given by

$$WL^0 = \int_{c/2}^{1} (2\theta - c)\, d\theta - \int_{\theta^{*0}}^{1} (2\theta - c)\, d\theta = 2\int_{0}^{c/2-\theta^{*0}} \theta\, d\theta.$$

Under market T, there will be insufficient entry in equilibrium, and the welfare loss in each period is given by

$$WL^T = \int_{c/2}^{1} (2\theta - c)\, d\theta - \int_{\theta^{*T}}^{1} (2\theta - c)\, d\theta = 2\int_{0}^{\theta^{*T}-c/2} \theta\, d\theta.$$

Therefore market 0 welfare dominates market T if and only if the equilibrium entry cutoff under market 0 is closer to the socially optimal entry cutoff $c/2$.

Step 2: There exists a unique $c^* \in (c^T, c^0)$ such that $c/2 - \theta^{*0} < \theta^{*T} - c/2$ if and only if $c > c^*$.

Define $\xi(c) = (\theta^{*T} - c/2) - (c/2 - \theta^{*0}) = \theta^{*T} + \theta^{*0} - c$.

From entry condition (A.11), we have

$$\frac{d\Pi^k(\theta^{*k}, \theta^{*k})}{d\theta^{*k}} \cdot \frac{d\theta^{*k}}{dc} = 1.$$

To simplify notation, let $Z_k(\theta^{*k}, \delta) = 1/(\frac{d\Pi^k}{d\theta^{*k}})$, $k \in \{0, T\}$. Thus $d\xi(c)/dc = Z_0 + Z_T - 1$. We need to show that $d\xi(c)/dc \geq 0$, which suffices to show that $Z_0 + Z_T - 1 \geq 0$.

From (A.11), we have

$$\text{(A.13)} \qquad \begin{aligned} \frac{\partial Z_0(\theta^*, \delta)}{\partial \theta^*} &= \frac{3(1+\theta^*)(\theta^*-3)N_0}{D_0} \\ \frac{\partial Z_T(\theta^*, \delta)}{\partial \theta^*} &= \frac{12(1+\theta^*)(\theta^*-3)N_T}{D_T}, \end{aligned}$$

where

$$N_0 = -1 - 3\theta^* - 25\delta - 3\delta\theta^{*2} + 33\delta\theta^* - 3\theta^{*2} - \theta^{*3} + 3\delta\theta^{*3},$$

$$D_0 = (\theta^{*4} - 4\theta^{*3} - 3\theta^{*2} + 3\delta\theta^{*2} - 2\delta\theta^* + 10\theta^* + 8 + 11\delta)^2,$$

$$N_T = -1 - 3\theta^* - 12\delta + 18\delta\theta^* - 3\theta^{*2} - \theta^{*3} + 2\delta\theta^{*3} \text{ , and}$$

$$D_T = (1+\delta)(-2\theta^{*4} + \delta\theta^{*4} + 8\theta^{*3} - 4\delta\theta^{*3} + 6\theta^{*2} - 6\delta\theta^{*2} - 20\theta^* + 12\delta\theta^* - 16 - 3\delta)^2.$$

Therefore the sign of $\partial Z_k/\partial\theta^{*k}$ is the opposite of the sign of $N_k, k \in \{0, T\}$. We want to show that $\partial Z_k/\partial\theta^{*k} \geq 0$. It suffices to show that $N_k \leq 0$. Since $N_0 - N_T = \delta(-13 - 3\theta^{*2} + 15\theta^* + \theta^{*3}) \leq 0$, it thus suffices to show that $N_T \leq 0$.

Note that $\frac{\partial N_T}{\partial \delta} = -12 + 18\theta^* + 2\theta^{*2}$ is strictly increasing in θ^*, and that $\frac{\partial N_T}{\partial \delta} = 0$ at $\theta^* = 3^{2/3} - 3^{1/3} \equiv \theta^{**}$. Therefore when $\theta^* \leq \theta^{**}$, $\frac{\partial N_T}{\partial \delta} \leq 0$, thus $N_T(\theta^*, \delta) \leq N_T(\theta^*, 0) = -1 - 3\theta^* - 3\theta^{*2} - \theta^{*3} < 0$; when $\theta^* > \theta^{**}$, $\frac{\partial N_T}{\partial \delta} > 0$, thus $N_T(\theta^*, \delta) \leq N_T(\theta^*, 1) = -13 + 15\theta^* - 3\theta^{*2} + \theta^{*3} \leq 0$. In both cases, $N_T \leq 0$ and hence $\partial Z_k/\partial\theta^{*k} \geq 0$ for $k \in \{0, T\}$. We thus have

$$\begin{aligned} Z_0(\theta^{*0}, \delta) \geq Z_0(0, \delta) &= \frac{27}{2(8+11\delta)} \\ Z_T(\theta^{*T}, \delta) \geq Z_T(0, \delta) &= \frac{27}{(1+\delta)(16+3\delta)}. \end{aligned}$$

Therefore,

$$\xi'(c) = Z_0 + Z_1 - 1 \geq \frac{27(32 + 41\delta + 3\delta^2)}{2(8+11\delta)(1+\delta)(16+3\delta)} > 0.$$

Since $\xi(c^T) = -[\frac{c^T}{2} - \theta^{*0}] < 0$ and $\xi(c^0) = [\theta^{*T} - \frac{c^0}{2}] > 0$, there exists a unique $c^* \in (c^T, c^0)$ such that $\xi(c^*) = 0, \xi(c) > 0$ when $c > c^*$, and $\xi(c) < 0$ when $c < c^*$.

Combining each of these pieces, we obtain the welfare ranking:

$c \leq \underline{c}$ or $c \geq \bar{c}$: Either no agents enter ($c \geq \bar{c}$) or all agents enter the market ($c \leq \underline{c}$) irrespective of the trademark trade rule, so no welfare effect is observed.

$c \in (\underline{c}, c^*)$: Permitting trademark sales discourages entry and increases welfare.

$c \in (c^*, \bar{c})$: Additional entry raises welfare. Such entry can be generated by banning trademark sales. ■

LEMMA 1. *In any equilibrium that satisfies conditions* ($\mathcal{A}$), ($\mathcal{M}$), *and* ($\mathcal{N}$), *only successful trademarks will be traded at positive prices, and the payment to agents with a successful trademark in each period* $t, t \geq 2$ *does not depend on the exact history of the trademark. That is*,

$$w_t(S) = w_t(SS) = w_t(SSS) = \cdots \equiv w_t(\mathbb{S}).$$

PROOF. First, due to ($\mathcal{A}$), it is apparent that only successful marks can be traded at positive prices in equilibrium.

Consider a new agent of type θ who chooses not to purchase a successful mark at $t, t \geq 1$. Its expected lifetime payoff is given by

$$\pi^\theta(N) = w_t(N) + \delta[\theta w_{t+1}(S) + (1-\theta)w_{t+1}(N)] + \delta^2\big[\theta^2 v_{t+2}(SS) + (1-\theta)\theta v_{t+2}(S)\big].$$

In writing down the above equation, we have made use of condition ($\mathcal{A}$), which implies that when an F is realized, the agent will change its mark to an N. Similarly, the agent's net expected lifetime payoff if it buys an S mark at t is given by

$$\pi^\theta(S) = -v_t(S) + w_t(S) + \delta[\theta w_{t+1}(SS) + (1-\theta)w_{t+1}(N)] + \delta^2\big[\theta^2 v_{t+2}(SSS) + (1-\theta)\theta v_{t+2}(S)\big].$$

Thus the payoff difference of a type θ agent from purchasing an S mark versus not purchasing one at t is given by $\Delta\pi^\theta = \pi^\theta(S) - \pi^\theta(N)$. Consider two types $\theta > \theta'$ of new agents, and compare their payoff differences as follows:

$$\Delta\pi^\theta - \Delta\pi^{\theta'} = \delta(\theta - \theta')[w_{t+1}(SS) - w_{t+1}(S)] + \delta^2(\theta^2 - \theta'^2)[v_{t+2}(SSS) - v_{t+2}(SS)] \geq 0. \tag{A.14}$$

The inequality above is due to condition ($\mathcal{M}$). Since $\Delta\pi^\theta - \Delta\pi^{\theta'}$ is a measure of incentive difference between two types in purchasing S mark, inequality (A.14) implies that a higher-type (new) agent has at least the same incentive as a lower-type (new) agent in purchasing an S mark.

Similarly, we can also compare the difference in the expected (future) payoffs in purchasing S marks between different types of continuing agents (who failed in the previous period). For $\theta > \theta'$, it can be verified that

$$\Delta\pi^{\theta} - \Delta\pi^{\theta'} = \delta(\theta - \theta')(v_{t+1}(SS) - v_{t+1}(S)) \geq 0. \tag{A.15}$$

Again the above inequality is due to ($\mathcal{M}$). Equations (A.14) and (A.15) thus jointly imply that higher-type agents (regardless of new or continuing) have (weakly) larger incentive in purchasing S marks. We further claim that $w_{t+1}(SS) = w_{t+1}(S)$. Suppose not; then the inequality in (A.14) must be strict, which implies that for new agents, higher-type agents have strictly larger incentive to purchase S marks. This, combined with the weak inequality in (A.15) and the scarcity of successful marks, implies that higher-type sorting must occur (at least among new agents) in purchasing S marks, which contradicts condition ($\mathcal{N}$).

By comparing the incentives in purchasing any other given successful mark $S\cdots S$, we can analogously show that $w_{t+1}(S\cdots SS) = w_{t+1}(S)$. Thus for any $t, t \geq 2$, we have

$$w_t(S) = w_t(SS) = w_t(SSS) = \cdots \equiv w_t(\mathcal{S}). \qquad \blacksquare$$

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