RAND Journal of Economics
Vol. 39, No. 2, Summer 2008
pp. 547–564

Search with learning: understanding asymmetric price adjustments

Huanxing Yang

and

Lixin Ye*

In many retail markets, prices rise faster than they fall. We develop a model of search with learning to explain this phenomenon of asymmetric price adjustments. By extending our static game analysis to the dynamic setting, we demonstrate that asymmetric price adjustments arise naturally. When a positive cost shock occurs, all the searchers immediately learn the true state; the search intensity, and hence the prices, fully adjust in the next period. When a negative cost shock occurs, it takes longer for nonsearchers to learn the true state, and the search intensity increases gradually, leading to slow falling of prices.

1. Introduction

■ Firms are quick to raise prices in response to their cost increases, but not so keen to reduce prices when their costs fall. This widespread phenomenon is known as asymmetric price adjustment, or the *rockets* and *feathers*. This pattern of asymmetric price adjustment has been reported in a broad range of product markets. In fact, a growing empirical literature documents asymmetric price adjustment in various markets, including gasoline (Bacon, 1991; Karrenbrock, 1991; Duffy-Deno, 1996; Borenstein et al., 1997; Eckert, 2002; Deltas, 2004), fruit and vegetables (Pick et al., 1991; Ward, 1982), beef and pork (Boyd and Brorsen, 1988; Goodwin and Holt, 1999; Goodwin and Harper, 2000), and banking (Hannan and Berger, 1991; Neumark and Sharpe, 1992; O'Brien, 2000).[1] According to Peltzman (2000), asymmetric price adjustment is found in more than two of every three markets examined in a large sample with 77 consumer goods and 165 producer goods.

Despite these extensive empirical studies confirming the general pattern of asymmetric price adjustment, there is little theoretical work examining this phenomenon. In fact, asymmetric price adjustment first appeared to be inconsistent with conventional microeconomic theory, which

*The Ohio State University; huanxing@econ.ohio-state.edu, lixinye@econ.ohio-state.edu.
We thank Matt Lewis, Howard Marvel, James Peck, and the seminar participants at the Third Summer Workshop on Industrial Organization and Management Strategy (2006, Beijing), the Midwest Economic Theory Meetings (Fall 2006, Purdue), and the 5th Annual International Industrial Organization Conference (May 2007, Savannah) for valuable comments and suggestions. We also benefited from insightful comments and suggestions from two anonymous referees. All remaining errors are our own.

[1] It is found that deposit rates respond more quickly to an increase than to a decrease of money market rates.

usually suggests that an increase or decrease of input prices should affect marginal costs, and hence move prices up or down in a symmetric, rather than asymmetric, way. As Peltzman (2000) puts it, the "stylized fact" of asymmetric price adjustment "poses a challenge to theory." This article attempts to help bridge such a gap in the literature.

More specifically, we develop a model of search with learning in a dynamic framework. We start with a description of the static game. There are a continuum of consumers and a continuum of firms with capacity constraints. Firms have a common unit production cost (either high or low). Although known to the firms, the cost is unknown to the consumers. There are three types of consumers: the low search cost consumers who always search, the high search cost consumers who never search, and critical consumers whose search cost is intermediate. The decision for a critical consumer to search or not depends on whether the expected benefit of searching outweighs her search cost, so the percentage of consumers who search (the search intensity) will be endogenously determined. We adopt the protocol of nonsequential search, that is, consumers who search observe the prices charged by all firms, so searchers always shop at firms with the lowest price available (unless they are rationed due to firms' capacity constraint, in which case they will shop at the firms with the next lowest price, and so forth). On the other hand, nonsearchers shop randomly and only observe one price.

In the static game, we show that there is a unique equilibrium. Critical consumers hold heterogeneous beliefs regarding the firms' production cost (the state), and the equilibrium search intensity only depends on critical consumers' distribution of initial beliefs. As more consumers' initial beliefs about the high-cost state lie below some cutoff level, the equilibrium search intensity increases. This is because prices are more dispersed when the cost is low due to competition among firms, leading to a higher expected gain from search. The equilibrium price distribution depends on the search intensity and the actual cost state. Specifically, the equilibrium prices are increasing in the actual cost, and are decreasing in search intensity, because each firm's demand becomes more elastic as more consumers search. Thus, the full adjustment of equilibrium prices requires the adjustment of search intensity, which solely depends on the critical consumers' belief-updating process.

We then extend our static game analysis to a dynamic setting where the cost evolves according to a Markov process with positive persistence. Because consumers never observe the cost realizations, each consumer updates her belief based on the history of prices she observed. Thus consumers have heterogeneous beliefs. In equilibrium, searchers and nonsearchers have different belief-updating processes. Searchers always correctly learn the true state, because they always observe the lowest price that fully reveals the true state. But nonsearchers do not always learn the true state.

Asymmetric price adjustment thus arises naturally. In the event of positive cost shocks, all the searchers among the critical consumers immediately learn the true state and stop searching. In the following period, no critical consumers search and the search intensity is the lowest possible. Thus, the search intensity and hence the prices fully adjust within two periods. In the event of negative cost shocks, it takes longer for critical consumers who do not search originally to learn the true state and start searching, thus the search intensity increases gradually, leading to slow falling of prices. To sum up, asymmetric price adjustment is caused by learning asymmetry between searchers and nonsearchers, which is closely related to the evolution of search intensity.

More formally, we show that given the evolution of the underlying cost states, there is a unique equilibrium in the dynamic game, with the evolution of the distribution of beliefs, the search intensity, and the prices uniquely determined. We demonstrate that as long as the cost shocks are persistent, the pattern of asymmetric price adjustments emerges in statistical sense on the equilibrium path of the dynamic game. Moreover, as the cost shocks become more persistent, the pattern of asymmetric price adjustments becomes more prominent, because the downward price adjustment on average spreads over longer periods of time.

Several recent papers (Lewis, 2005; Tappata, 2006; Cabral and Fishman, 2006) also attempt to explain asymmetric price adjustment based on search models.[2] Lewis (2005) develops a reference price search model in which the expected distribution of prices is exogenously given, rather than endogenously determined; consumers have adaptive expectations and thus are not rational. On the other hand, consumers are rational in our model, because they form expectations of prices based on all the information available.

Cabral and Fishman (2006) develop a search model in which the cost changes are positively but not perfectly correlated across firms.[3] They show that consumers have a greater incentive to search in the case of large price increases or small price decreases, but little incentive to search when prices increase a little or decrease by a lot. This implies that firms are reluctant to change prices when costs decrease by a little bit or increase by a lot, but quick to change prices as costs increase by a little bit or decrease by a lot. In other words, when the cost change is small, the price adjustment exhibits downward rigidity and upward flexibility; when the cost change is big, the asymmetry is reversed: prices exhibit downward flexibility and upward rigidity. These implications are quite different from ours.

The paper that is most closely related to ours is Tappata (2006). Our article differs from Tappata in an important aspect in terms of modelling. That is, although Tappata assumes that the firms' past costs are known to the consumers, we do not impose this assumption in our analysis. Because consumers know past costs, there is no learning in Tappata. In contrast, the learning asymmetry between searchers and nonsearchers about the underlying cost is the driving force in our analysis. This modelling difference leads to very different empirical implications. In Tappata's setting, because there is no learning, it takes exactly two periods for prices to fully adjust to both the positive and negative cost shocks. Moreover, in Tappata, the asymmetry in price adjustments is only present in the first period after a cost shock occurs: the magnitude of price adjustment in the first period is bigger in the case of positive cost shocks than in the case of negative cost shocks. In contrast, by endogenizing the time periods that are needed for prices to fully adjust to cost shocks, we are able to show that asymmetric price adjustment goes beyond the first period after a cost shock occurs: although it takes two periods for prices to fully adjust to positive cost shocks, it takes much longer periods for prices to fully adjust in response to negative cost shocks.

Our article also contributes to the literature on consumer search, in that we develop a dynamic search model with consumers, in a heterogeneous fashion, learning about the underlying states based on the personal histories of prices they observed.[4] Several previous papers have studied equilibrium search with learning (Benabou and Gertner, 1993; Dana, 1994; Fishman, 1996). Benabou and Gertner (1993) study how the correlations among firms' cost shocks affect consumers' incentive to search and the equilibrium prices. In a static model, Dana (1994) shows that if consumers are uncertain about firms' costs, then the response of prices to cost shocks will be limited. In a dynamic framework, Fishman (1996) shows that cost shocks have different short-run and long-run effects on prices.[5] None of these papers study asymmetric price adjustments.

The article is organized as follows. Section 2 presents the static game and characterizes the unique static game equilibrium. In Section 3, we extend the static game analysis to the dynamic setting and show that asymmetric price adjustment arises naturally on the equilibrium path. In

[2] Borenstein et al. (1997) propose an explanation for asymmetric price adjustments based on Rotemberg and Saloner's (1986) model of tacit collusion with stochastic shocks.

[3] More specifically, they consider two firms. The costs of the firms either both increase or both decrease, although the magnitude of the changes might be different.

[4] For models based on nonsequential search, see, for example, Salop and Stiglitz (1977); Braverman (1980); and Varian (1980). For models based on sequential search, see, for example, Burdett and Judd (1983); Rob (1985); and Stahl (1989).

[5] More specifically, Fishman shows that in the case of a general cost (common to all firms) increase, consumers search too much so as to limit the extent to which prices increase in the short run; however, in the case of an idiosyncratic cost (specific to only one firm) increase, consumers search too little, leading to price overshooting.

Section 4, we discuss the restrictions of our key assumptions and the robustness of our results. Section 5 concludes.

2. Static game

■ **The model.** We consider a market with a continuum of firms producing a homogeneous good. The total measure of firms is normalized to be 1. All the firms have the same cost c in producing each unit of the good (firms have common cost shocks). *Ex ante,* c (i.e., the state of the world) can take value either c_H or c_L, where $c_L < c_H$. At the beginning of the period, firms observe the realization of the cost and then compete in prices. We also assume that each firm has a capacity constraint k (finite), that is, no firm can sell more than k units of the good.[6]

There is a continuum of consumers with total measure $\beta > 1$. The parameter β can also be interpreted as the number of consumers per firm in the market. Each consumer has a unit demand with a choke price of $v > c_H$. We assume that $\beta < k$, that is, the number of consumers per firm in the market is less than each firm's capacity constraint.[7] Consumers do not observe the realization of c. Instead, consumers hold beliefs about the cost realization, which might be heterogeneous among consumers. Let α denote a consumer's belief if she believes that the probability of $c = c_H$ is α. The distribution of beliefs among consumers will be specified later. Before observing prices, consumers make decisions regarding whether to search (become informed) or not to search (stay uninformed). We adopt the protocol of nonsequential search. Informed consumers observe all the realized prices and purchase from the firms (stores) with the lowest price available.[8] Each uninformed consumer shops randomly at a firm (store) and only observes that firm's price.

Each consumer's type is characterized by her search cost. The first type of consumer (with proportion λ_1) each has search cost $s_L = 0$. These consumers are also called *shoppers*, who can be interpreted as those who have obtained price information without incurring nontrivial search cost (e.g., from TV or internet advertisements, etc.). This type of consumer always searches in equilibrium regardless of their beliefs about the underlying state. The second type of consumer (with proportion λ_2) each has search cost s_H. We assume that $s_H > v$ so that this type of consumer never searches (regardless of their beliefs about the underlying state). The rest of the consumers (with proportion $1 - \lambda_1 - \lambda_2$) each have intermediate search cost $s_M \in (s_L, s_H)$ and they may or may not search depending on their beliefs about the underlying state. Because beliefs about the underlying state only matter for this type of consumer, they are henceforth referred to as the *critical consumers*. Let $F(\alpha)$ denote the cumulative distribution function of the beliefs among the critical consumers. In other words, $F(\alpha)$ is the fraction of the critical consumers whose beliefs are lower than α.

Because there is a capacity constraint for each firm, rationing may occur: for a low-price firm, the number of consumers shopping at this firm may be bigger than k. We adopt the proportional rationing rule: if rationing occurs at a firm, each consumer (nonsearcher or searcher) who shops at that firm will be able to purchase a unit of the good with the same probability. If a nonsearcher is rationed, she shops randomly at other firms without incurring any cost. If a searcher is rationed, she goes to the firm with the lowest price among the remaining firms. Should rationing also occur there, the same search procedure applies until the searcher purchases a unit of the good.

---

[6] In the dynamic setting, this assumption implies that no firm can sell more than k units of the good per period. We believe that capacity constraint is prevalent in many product markets. For example, sales of perishable goods such as fruit, vegetables, beef, or pork are often constrained by the storage space for a given period; even the supply capacity of gasoline stations within a given period can be limited by things such as petroleum pipeline systems, which are often operating at their full capacities. This being said, we impose the capacity constraint mainly for tractability of equilibrium analysis, which is further discussed in Section 4.

[7] As will become clear, this assumption makes competition among firms nontrivial.

[8] The assumption that searchers observe all the realized prices follows Varian (1980). In Burdett and Judd (1983), searchers are only able to observe a subset of realized prices. This alternative setting is further discussed in Section 4.

The timeline of the game is as follows. First, the production cost (the state of the world) is realized and all the firms observe the common state. The firms then simultaneously set prices. Finally, consumers decide whether to search and make purchases accordingly.[9]

As is standard in the search literature, we focus on symmetric equilibria in which all firms employ the same pricing strategies. Let G be the price distribution and μ be the proportion of informed consumers, or the search intensity, which is endogenously determined in our model ($\mu \geq \lambda_1$). Given the distribution of critical consumers' beliefs $F(\alpha)$, a symmetric perfect Bayesian equilibrium is characterized by a pair (μ^*, $G^*(\cdot \mid c)$), with the following properties: given the equilibrium search intensity μ^*, firms' optimal pricing strategies yield the equilibrium price distribution $G^*(\cdot \mid c)$; given $G^*(\cdot \mid c)$ and $F(\alpha)$, consumers' optimal search decisions give rise to the equilibrium search intensity μ^*.

□ **Analysis with fixed search intensity.** We first derive the equilibrium price distribution given the search intensity μ and the production cost c (the state). Let $\underline{p}$ be the lowest price charged in equilibrium, and let the proportion of the firms that charge $\underline{p}$ be $\eta(\underline{p})$. Note that a $\underline{p}$ firm's sales are

$$\min\left\{\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta, k\right\}.$$

The first term in the bracket is the demand for a $\underline{p}$ firm: it attracts $(1-\mu)\beta$ nonsearchers, and gets $\frac{\mu\beta}{\eta(\underline{p})}$ searchers. The quantity a $\underline{p}$ firm sells is simply the minimum of its demand and capacity.

Lemma 1. In any equilibrium, a firm that charges $\underline{p}$ must sell k units of the good.

Proof. Suppose in negation, a firm charging $\underline{p}$ sells strictly less than k units in equilibrium. Then by undercutting $\underline{p}$ by an arbitrarily small amount ε, this firm can attract a positive measure of searchers, thus increasing its sales to k without affecting the profit margin, which destroys the proposed equilibrium. This implies that in any equilibrium, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \geq k$.

Lemma 2. There is no equilibrium in which prices are continuously distributed on $[\underline{p}, \bar{p}]$, for any $\bar{p}$ such that $\underline{p} < \bar{p} \leq v$.

Proof. Consider a candidate equilibrium in which prices are continuously distributed on $[\underline{p}, \bar{p}]$ for some $\bar{p}$ such that $\underline{p} < \bar{p} \leq v$. A necessary condition for this to be an equilibrium is that consumers should be rationed at any $p \in (\underline{p}, \bar{p})$: if consumers are not rationed at such a p, then there is no point to charge $p + \varepsilon$, because a firm can only attract nonsearchers in that case and its demand is given by $(1-\mu)\beta$, which is strictly dominated by charging v. But given that consumers are rationed at any $p \in (\underline{p}, \bar{p})$, which means that each firm charging any $p \in (\underline{p}, \bar{p})$ sells k, a $\underline{p}$ firm can increase its profit margin without affecting its sales by deviating to $p \in (\underline{p}, \bar{p})$. This destroys the proposed equilibrium.

Lemma 3. In any equilibrium, consumers shopping at any $\underline{p}$ firm are not rationed. That is, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \leq k$.

Proof. Suppose in negation, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta > k$. In this case, a positive measure of consumers (hence searchers) are rationed at $\underline{p}$ firms. By Lemma 2, there is a positive number ε such that no firm charges at any price $p \in (\underline{p}, \underline{p} + \varepsilon)$. Then a firm who charges $\underline{p}$ can deviate to some price $p' \in (\underline{p}, \underline{p} + \varepsilon)$. Under this deviation, this firm can still attract enough searchers (because a positive measure of searchers are rationed at $\underline{p}$ firms), and thus sell k units with an increased profit margin. This destroys the proposed equilibrium.

Therefore, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \leq k$ in equilibrium, if there is any. This implies that searchers are not rationed at $\underline{p}$ firms.

[9] Because the first and the second type consumers either always or never search, only the critical consumers have nontrivial decisions to make.

Lemma 1 and Lemma 3 jointly imply that in equilibrium, we must have $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta = k$. That is, no rationing occurs at firms charging the lowest price. Next we will establish that any equilibrium must have a two-point price distribution.

Lemma 4. Given μ and c, in any candidate equilibrium each firm must charge either $p = v$ or $p = \underline{p} \in (c, v)$($\underline{p}$ is to be determined).

Proof. Suppose there is an equilibrium price strictly between $\underline{p}$ and v. Then a firm that charges this price can increase its profit by deviating to charging v. This deviation leads to a higher profit margin per unit of sales, with the demand unchanged (only the lowest price can attract informed consumers, given that they are not rationed at the lowest price by Lemma 3). So the only possible equilibrium is either all firms charging the same price, or a two-price distribution on $\underline{p}$ and v.

First consider the candidate equilibrium in which firms charge the same price $\bar{p} = \underline{p} > c$. The equilibrium demand for each firm is β. But then a firm can undercut p a little bit and sell $k > \beta$ without affecting the profit margin per unit of sales. Thus, this type of equilibrium cannot exist. All firms charging $p = c$ cannot be an equilibrium either, because by deviating to $p = v$ a firm can get a positive profit by selling to uninformed consumers ($s_H > v$ means that a measure of $\lambda_2\beta$ consumers are always uninformed).

Now the only candidate equilibrium left is that firms charge either v or $\underline{p}$. Clearly, $\underline{p} < v$. What remains to be shown is that $\underline{p} > c$. Because charging v yields a positive profit, charging $\underline{p}$ should also yield a positive profit, which implies $\underline{p} > c$.

By the above lemmas, there is only one possible equilibrium, with equilibrium prices characterized by a two-point distribution. Denote $\pi(v)$ and $\pi(\underline{p})$ as the profits of a firm that charges v and $\underline{p}$, respectively. Explicitly,

$$\pi(v) = (1-\mu)\beta(v-c)$$
$$\pi(\underline{p}) = k(\underline{p}-c).$$

By Lemma 3, a firm charging v can only attract nonsearchers. Thus its demand and sales are $(1-\mu)\beta$, and its profit margin is $v - c$.[10] By Lemma 1, a firm charging $\underline{p}$ sells k, with a profit margin $\underline{p} - c$. The equilibrium is characterized by the following two conditions:

$$\pi(v) = \pi(\underline{p}) \tag{1}$$

$$\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta = k. \tag{2}$$

Condition (1) says that a firm should be indifferent between charging v and $\underline{p}$, and condition (2) says that the demand for a $\underline{p}$ firm exactly equals its capacity k.

Proposition 1. Given μ and c, there is a unique equilibrium: a proportion of $1 - \eta(\underline{p})$ firms charge price v, and a proportion of $\eta(\underline{p})$ firms charge price $\underline{p} \in (c, v)$, where $\eta(\underline{p})$ and $\underline{p}$ are determined by conditions (1) and (2). More explicitly,

$$\eta(\underline{p}) = \frac{\mu\beta}{k-\beta+\mu\beta} \tag{3}$$

$$\underline{p} = c + \frac{(1-\mu)\beta}{k}(v-c). \tag{4}$$

Proof. We only need to show the price distribution specified above is an equilibrium. Note that from condition (2), searchers are not rationed at $\underline{p}$. We show that firms have no incentive to deviate. First consider a firm charging v. Deviating to any $p \in (\underline{p}, v)$ would lead to a lower profit margin without increasing the sales (because searchers are not rationed at $\underline{p}$), hence such a deviation is

[10] Note that $(1-\mu)\beta < k$ because $\beta < k$.

not profitable. Deviating to $\underline{p}$ yields the same profit by condition (1). Deviating to $p < \underline{p}$ is strictly dominated by charging $\underline{p}$, because a firm cannot sell more than k. Thus a firm charging v has no incentive to deviate. Next consider a firm charging $\underline{p}$. By a similar argument, the firm has no incentive to deviate to $p < \underline{p}$. If the firm deviates to $p \in (\underline{p}, v]$, it only attracts nonsearchers and hence sells $(1 - \mu)\beta$ only, because searchers are not rationed at $\underline{p}$.[11] Thus the most profitable deviation is to set the price at v, which, by condition (1) yields the same profit as no deviation. Thus, a firm charging $\underline{p}$ has no incentive to deviate either. Solving (1) and (2) yields expressions (3) and (4).

Note that in this unique equilibrium, no consumer is rationed: the demand for a $\underline{p}$ firm exactly equals its capacity k, and the demand for a v firm is strictly less than k. From (3), we can see that $\eta(\underline{p})$ is increasing in the search intensity μ. Intuitively, as μ increases the demand becomes more elastic, and more firms charge lower price. Moreover, $\eta(\underline{p})$ does not depend on c directly. By (4), $\underline{p}$ is increasing in c and decreasing in μ. As demand becomes more elastic (μ increases), charging the lower price becomes relatively more profitable, other things equal. To restore the indifference condition, the lower price must decrease to reduce the profit margin for the lower-price firms. Note that the presence of the capacity constraint keeps the profit margin of $\underline{p}$ firms positive, by restricting the competition among $\underline{p}$ firms.

□ **Equilibrium with endogenously determined search intensity.** Now we analyze the equilibrium of the game, with search intensity μ endogenously determined. We assume that firms know the distribution of beliefs $F(\alpha)$, while consumers may not know $F(\alpha)$.[12] Recall that the cost c can only take two possible values c_H and c_L, which is the same among all the firms. Proposition 1 characterizes firms' equilibrium price distribution given any search intensity μ. The remaining task is to derive the equilibrium search intensity μ^*. We first compute the expected gain from searching given the equilibrium price distribution, μ, and a consumer's belief, α:

$$
\begin{aligned}
E[p - \underline{p} \mid \alpha] &= \alpha[(1 - \eta(\underline{p}_H))v + \eta(\underline{p}_H)\underline{p}_H - \underline{p}_H] + (1 - \alpha)[(1 - \eta(\underline{p}_L))v + \eta(\underline{p}_L)\underline{p}_L - \underline{p}_L] \\
&= (1 - \eta(\underline{p}_i))[\alpha(v - \underline{p}_H) + (1 - \alpha)(v - \underline{p}_L)] \\
&= \frac{k - \beta}{k}\{v - [\alpha c_H + (1 - \alpha)c_L]\}, \qquad (5)
\end{aligned}
$$

where $\underline{p}_H$ denotes $\underline{p}(c_H)$ and $\underline{p}_L$ denotes $\underline{p}(c_L)$. Note that by (4), $\underline{p}_H > \underline{p}_L$ given μ.

From (5) we can see that $E[p - \underline{p} \mid \alpha]$ does not depend on μ. Thus, search does not exhibit complementarity. This is because an increase in μ has two countervailing effects. The first effect is that $\underline{p}$ decreases as μ increases, which increases the return of search. On the other hand, an increase in μ causes more firms to charge $\underline{p}$ ($\eta(\underline{p})$ increases), which reduces the average price and raises the payoff of a nonsearcher. These two effects exactly offset each other, as shown by (5).

From (5), we see that the expected gain from search is decreasing in α for critical consumers. Thus, there is a cutoff belief $\widehat{\alpha}$ such that all critical consumers with beliefs below $\widehat{\alpha}$ search and those above $\widehat{\alpha}$ do not search. If $s_M \geq \frac{k-\beta}{k}(v - c_L)$, then even the consumer with the most optimistic belief ($\alpha = 0$) cannot afford the search, so $\widehat{\alpha} = 0$; on the other hand, if $s_M \leq \frac{k-\beta}{k}(v - c_H)$, then even the consumer with the most pessimistic belief ($\alpha = 1$) can afford the search, so $\widehat{\alpha} = 1$. In what follows we will focus on the most interesting case in which

[11] A single firm deviation would not affect the total measure of the firms who set price $\underline{p}$ (due to our assumption of continuum of firms). Thus, condition (2) implies that the following condition continues to hold even after one single firm deviates:

$$\frac{\mu\beta}{\text{Measure of firms setting } \underline{p}} + (1 - \mu)\beta = k,$$

which in turn implies that searchers will not be rationed, even though one single firm deviates by setting a higher price (all before consumers move, as specified in our timeline).

[12] These properties will be justified in the dynamic model.

$$\frac{k-\beta}{k}(v-c_H) < s_M < \frac{k-\beta}{k}(v-c_L). \tag{6}$$

In this case, $\widehat{\alpha}$ lies in (0,1), which is determined by the following indifference condition:

$$s_M = E[p - \underline{p} \mid \widehat{\alpha}] = \frac{k-\beta}{k}\left\{v - [\widehat{\alpha}c_H + (1-\widehat{\alpha})c_L]\right\}. \tag{7}$$

Solving (7) we have

$$\widehat{\alpha} = \frac{(v-c_L)(k-\beta) - ks_M}{(c_H - c_L)(k-\beta)}. \tag{8}$$

Then the equilibrium search intensity μ^* can be computed as follows:

$$\mu^* = \lambda_1 + (1 - \lambda_1 - \lambda_2)F(\widehat{\alpha}). \tag{9}$$

Note that condition (7) pins down a unique $\widehat{\alpha} \in (0, 1)$, and hence μ^* is also unique. Moreover, (8) shows that $\widehat{\alpha}$ is independent of all the endogenous variables, and thus is common knowledge. It is easily seen that, given $F(\alpha)$, μ^* and G^* $(\cdot \mid \mu^*, c)$ described by (3) and (4) constitute the unique equilibrium of the static game. Given $F(\alpha)$ and G^* $(\cdot \mid \mu^*, c)$, the optimal decision rules about search give rise to μ^*. Because firms know $F(\alpha)$, they can correctly anticipate μ^* by (9). And given μ^*, the firms' optimal pricing strategies result in the equilibrium price distribution G^* $(\cdot \mid \mu^*, c)$. Note that rationing does not occur in equilibrium. This is the case for two reasons. First, firms can correctly anticipate the equilibrium search intensity μ^*. Second, although $\underline{p}_H > \underline{p}_L$, $\eta(\underline{p}_H) = \eta(\underline{p}_L)$, that is, the proportion of the firms charging the lower price does not depend on the cost realization.

The equilibrium price distribution is determined by the cost realization and consumers' beliefs $F(\alpha)$. The average price is lower under state c_L than under state c_H. Moreover, the price distribution is more dispersed under state c_L (the gap between the average price and the lowest price is larger), which leads to a higher expected return to search. Thus, as more consumers' beliefs lie below $\widehat{\alpha}$ (or $F(\widehat{\alpha})$ increases), the equilibrium search intensity μ^* increases. As a result, both $\underline{p}_H$ and $\underline{p}_L$ are decreasing in $F(\widehat{\alpha})$, and $\eta(\underline{p}_H) = \eta(\underline{p}_L)$ are increasing in $F(\widehat{\alpha})$. Define the average equilibrium price as $\bar{p}(c_i, \mu^*)$:

$$\begin{aligned}\bar{p}(c_i, \mu^*) &= \eta(\underline{p}_i)\underline{p}_i + [1 - \eta(\underline{p}_i)]v \\ &= \frac{\mu^*\beta}{k-\beta+\mu^*\beta}\left[c + \frac{(1-\mu^*)\beta}{k}(v-c)\right] + \frac{k-\beta}{k-\beta+\mu^*\beta}v.\end{aligned} \tag{10}$$

It is easily seen from (10) that $\bar{p}(c_i, \mu^*)$ is increasing in c_i and decreasing in μ^*. Thus, the average price is also decreasing in $F(\widehat{\alpha})$. The following proposition summarizes these results.

Proposition 2. There is a unique equilibrium with the equilibrium search intensity given by (8) and (9). Moreover, μ^* is increasing in $F(\widehat{\alpha})$; both $\underline{p}_H$ and $\underline{p}_L$ are decreasing in $F(\widehat{\alpha})$, and both $\eta(\underline{p}_H)$ and $\eta(\underline{p}_L)$ are increasing in $F(\widehat{\alpha})$; $\bar{p}(c_i, \mu^*)$ is decreasing in μ^* and $F(\widehat{\alpha})$.

According to the previous analysis, changes in equilibrium price distribution can be decomposed into two components. The first component is the change resulting from changes in cost realization, and the second component is the change resulting from changes in consumers' search intensity, which is governed by the distribution of consumers' beliefs.

3. Dynamic model

■ We now extend our analysis to the dynamic setting, and endogenize critical consumers' beliefs. Time t is discrete and $t = 1, 2, \ldots$. In each period the static game is played. We assume that the common cost evolves according to a Markov process, with ρ being the persistence parameter. That is,

$$\Pr(c_{t+1} = c_H | c_t = c_H) = \Pr(c_{t+1} = c_L | c_t = c_L) = \rho,$$

where $\rho > 1/2$. At $t = 1$, the two cost states are equally likely. This Markov structure of cost evolution is common knowledge. Although firms always observe the cost state of the current period, consumers never observe past or current cost realizations. Instead, each consumer updates her belief about the cost based on the price history she observes. Tappata (2006) assumes that last period cost realization is observable to consumers, and thus prices adjust fully to cost shocks in two periods.[13]

From the static model, we see that the equilibrium price distribution in a particular period depends on consumers' beliefs and the cost realization. The key in our dynamic game analysis is to trace the belief-updating process among critical consumers. From the static model, the lower price $\underline{p}$ is responsive to cost realizations: $\underline{p}_H > \underline{p}_L$ for any given μ. To simplify our analysis, we make assumptions about the parameter values such that the lower bound of $\underline{p}_H$ (under the highest possible μ) is greater than the upper bound of $\underline{p}_L$ (under the lowest possible μ). Note that the highest possible μ is $1 - \lambda_2$ (all critical consumers search), and the lowest possible μ is λ_1 (no critical consumers searches). So in effect, we assume

$$\frac{c_H - c_L}{v - c_L} > \frac{\beta(1 - \lambda_1 - \lambda_2)}{k - \beta\lambda_2}, \tag{11}$$

which implies $\underline{p}_H(\mu = 1 - \lambda_2) > \underline{p}_L(\mu = \lambda_1)$, that is, there is no overlap between the supports of $\underline{p}_H$ and $\underline{p}_L$ in equilibria of the dynamic game (hence (11) can also be termed the *nonoverlapping condition*).

Given this nonoverlapping condition, a consumer who observes $\underline{p}$ can correctly infer the true cost of that period. As a result, her initial belief next period is either ρ or $1 - \rho$. On the other hand, the high price (v) is not responsive to cost realizations. Thus, if a consumer observes a price v, her belief about the true cost in the current period (α_t^p, superscript p denotes the posterior) is updated as follows (suppose her initial belief is that $\Pr(c_t = c_H) = \alpha_t$):

$$\alpha_t^p = \frac{\alpha_t(1 - \eta(\underline{p}_H))}{\alpha_t(1 - \eta(\underline{p}_H)) + (1 - \alpha_t)(1 - \eta(\underline{p}_L))} = \alpha_t,$$

because by (3), $\eta(\underline{p}_H) = \eta(\underline{p}_L)$ in any period. Hence, the consumer's belief is not updated at all. As a result, her initial belief about the cost for the next period will be

$$\alpha_{t+1} = \rho\alpha_t + (1 - \rho)(1 - \alpha_t). \tag{12}$$

Note that a consumer's initial belief has an upper bound ρ and a lower bound $1 - \rho$. Moreover, if a critical consumer with initial belief ρ observes v in all the subsequent n periods, then by (12) her initial belief converges to 1/2 from above. To further facilitate our analysis, in what follows we will assume that s_M has bounds tighter than those given in (6). That is,

$$\frac{k - \beta}{k}\left[(v - c_L) - \frac{1}{2}(c_H - c_L)\right] < s_M < \frac{k - \beta}{k}\left[(v - c_L) - (1 - \rho)(c_H - c_L)\right]. \tag{13}$$

Under this assumption, it can be verified that $\widehat{\alpha} \in (1 - \rho, 1/2)$. The property that $\widehat{\alpha} > 1 - \rho$ ensures that critical consumers search under the most optimistic belief; otherwise, they will not search regardless of their beliefs. The property that $\widehat{\alpha} < 1/2$ ensures that critical consumers with initial belief ρ who observe v in all the subsequent periods will not search. Although not affecting the robustness of our result, assumption (13) simplifies our analysis because we do not need to trace the beliefs of those consumers who do not search.

From the above discussion, we can see that searchers and nonsearchers have different belief-updating processes. Because a searcher always observes the low price, she can infer the true cost state correctly. However, a nonsearcher only observes one price, and this price may be the high

[13] Because the past cost history is assumed to be known to the consumers, there is no learning in Tappata's model.

price v. In this case, her posterior is not updated. If she observes the lower price, she updates her posterior correctly. As we will see, the difference in belief-updating processes between searchers and nonsearchers is the key to understanding asymmetric price adjustments.

□ **Equilibrium.** By equation (8), $\widehat{\alpha}$ does not depend on μ_t^* (the equilibrium search intensity in period t) or the cost state in period t. Therefore, the distribution of beliefs in period t can be summarized by $F_t(\widehat{\alpha})$. Hence in each period, the state of the economy can be summarized by $F_t(\widehat{\alpha})$ and c_t, and the equilibrium in the dynamic game can be characterized by the sequence of $\{c_t, F_t(\widehat{\alpha}), \mu_t^*, G^*(\cdot \mid \mu_t^*, c_t)\}$.

Proposition 3. The equilibrium in the dynamic game exists and is unique for any evolution of $\{c_t\}$.

Proof. Suppose firms correctly anticipate $F_t(\widehat{\alpha})$. Then the equilibrium μ_t^* is determined by

$$\mu_t^* = \lambda_1 + (1 - \lambda_1 - \lambda_2)F_t(\widehat{\alpha}). \tag{14}$$

From the analysis of the static game, the equilibrium price distribution in period t, $G^*(\cdot \mid \mu_t^*, c_t)$, is uniquely determined. The existence of equilibrium in the dynamic game boils down to the following condition: firms are able to anticipate $F_t(\widehat{\alpha})$ correctly. The uniqueness of equilibrium is guaranteed if the evolution of $F_t(\widehat{\alpha})$ is unique given $\{c_t\}$. Below we will show that these two properties hold.

First, we show that firms can infer $F_1(\widehat{\alpha})$. In period $t = 1$ all critical consumers hold the same initial belief $1/2$ (because there is no prior history of prices). Given $\widehat{\alpha} < 1/2$, $F_1(\widehat{\alpha}) = 0$, and no critical consumers search in period $t = 1$. Because it is common knowledge that all critical consumers hold the same initial belief $1/2$ in period 1, firms can correctly infer that $F_1(\widehat{\alpha}) = 0$.

Second, we show that a critical consumer's belief will never be in $(\widehat{\alpha}, 1/2)$. To see this, first note that each critical consumer's initial belief in period 1 is $1/2$. If a critical consumer observes a sequence of higher prices v, her initial belief remains $1/2$ by (12). Her belief will be different only if she observes the lower price in the last period. In that case, her initial belief will either be ρ or $1 - \rho$. If her belief is $1 - \rho$, then she will search and observe the lower price, and her next period belief will be either ρ or $1 - \rho$. If her belief is ρ hence she does not search, then it will either converge to $1/2$ from above if she keeps observing the high price, or be revised to ρ or $1 - \rho$ if she happens to observe the lower price in some period. Therefore, in all cases a critical consumer's belief is outside the range of $(\widehat{\alpha}, 1/2)$.

Third, we show that if the firms know $F_t(\widehat{\alpha})$, then combining this with the information about c_t, they can correctly infer $F_{t+1}(\widehat{\alpha})$. We discuss two cases in order.

In the first case, suppose $c_t = c_H$. For critical consumers whose $\alpha_t \le \widehat{\alpha}$, they will search in period t and learn the true state $c_t = c_H$; hence their initial belief in period $t + 1$ is ρ. For critical consumers whose $\alpha_t \in [1/2, \rho]$, they will not search in period t. Among those consumers, an $\eta_t(\underline{p}_H)$ portion observe $\underline{p}_H$ and learn the true state c_H. Thus their initial belief in period $t + 1$ becomes ρ. The remaining $1 - \eta_t(\underline{p}_H)$ portion of consumers observe v hence no belief updating occurs. Their initial beliefs in period $t + 1$ remain within $[1/2, \rho]$. Aggregating over all the critical consumers, we can see that $F_{t+1}(\widehat{\alpha}) = 0$. Thus, we have the following transition equation for the distribution of beliefs:

$$\text{if } c_t = c_H : F_{t+1}(\widehat{\alpha}) = 0 \text{ given any } F_t(\widehat{\alpha})\,. \tag{15}$$

In the second case, suppose $c_t = c_L$. For critical consumers whose $\alpha_t \le \widehat{\alpha}$, they will search in period t and learn the true state $c_t = c_L$; hence their initial belief in period $t + 1$ is $1 - \rho$. For critical consumers whose $\alpha_t \in [1/2, \rho]$, they will not search in period t. Among those consumers, an $\eta_t(\underline{p}_L)$ portion observe $\underline{p}_L$ and learn the true state c_L. Thus their initial belief in period $t + 1$ becomes $1 - \rho$. The remaining $1 - \eta_t(\underline{p}_H)$ portion of consumers observe v with no information updated. Their initial beliefs in period $t+1$ remain in $[1/2, \rho]$. Aggregating over all the critical consumers, we obtain another transition equation for the distribution of beliefs,

$$\text{if } c_t = c_L{:}F_{t+1}(\widehat{\alpha}) = F_t(\widehat{\alpha}) + [1 - F_t(\widehat{\alpha})]\eta_t(\underline{p}_L)$$
$$= F_t(\widehat{\alpha}) + \frac{\mu_t^* \beta}{k - \beta + \mu_t^* \beta}[1 - F_t(\widehat{\alpha})]. \tag{16}$$

From (15) and (16), we can see that given c_t and $F_t(\widehat{\alpha})$, $F_{t+1}(\widehat{\alpha})$ is uniquely determined. Moreover, firms can correctly anticipate $F_{t+1}(\widehat{\alpha})$ based on their information about c_t, $F_t(\widehat{\alpha})$, and μ_t^*.

Therefore, given the evolution of $\{c_t\}$, there is a unique equilibrium characterized by the sequence of $\{c_t, F_t(\widehat{\alpha}), \mu_t^*, G^*(\cdot \mid \mu_t^*, c_t)\}$ in the dynamic game.

Note that except for period 1, critical consumers' beliefs are heterogeneous due to the different price histories they experienced. Also note that we do not need to trace the evolution of the exact distribution of beliefs $F_t(\alpha)$, which would be cumbersome to describe. Instead, we only need to trace the evolution of $F_t(\widehat{\alpha})$ to determine the equilibrium search intensity μ_t^*. Another interesting property is that, whereas firms can correctly infer $F_t(\widehat{\alpha})$, μ_t^*, and $G^*(\cdot \mid \mu_t^*, c_t)$, consumers in period t do not need to hold correct beliefs about $F_t(\widehat{\alpha})$, μ_t^*, and $G^*(\cdot \mid \mu_t^*, c_t)$. The main reason is that consumers do not observe the history $\{c_t\}$; instead, they update their beliefs about $\{c_t\}$ and $G^*(\cdot \mid \mu_t^*, c_t)$ based on their personal histories of the prices they encountered.

□ **Asymmetric price adjustments.** According to the analysis of the static game, changes in equilibrium price distribution can be decomposed into two components, one due to the change in cost realization and the other due to the change in consumers' search intensity. In the dynamic game, the highest average price across periods (and the highest possible lower price across periods) arises when $c_t = c_H$ and $\mu_t^* = \underline{\mu} = \lambda_1$ (no critical consumers search). Note that this corresponds to the case where $c_t = c_H$ and consumers have full information about c_t. On the other hand, the lowest possible average price (and the lowest possible lower price) arises when $c = c_L$ and $\mu_t^* = \bar{\mu} = 1 - \lambda_2$ (all critical consumers search). Similarly, this corresponds to the case where $c = c_L$ and consumers have full information about c_t. Therefore, in state c_H we say price is fully adjusted if $\mu^* = \underline{\mu} = \lambda_1$, and in state c_L we say price is fully adjusted if $\mu^* = \bar{\mu} = 1 - \lambda_2$.

In the dynamic game, because consumers do not observe past cost realizations, their beliefs, hence the search intensity μ^*, do not adjust as quickly as the underlying cost state changes. More importantly, the speed of adjustments for the beliefs and search intensity is different under positive cost shocks and negative cost shocks. Because the average price and the lower price move in the same direction, for brevity of exposition in what follows they are often simply referred to as *the prices*. The following propositions identify the asymmetry in price adjustments if the cost state persists after a shock occurs.

Proposition 4. Suppose a positive cost shock occurs in period $t+1$. Then regardless of $F_{t+1}(\widehat{\alpha})$, $F_{t+2}(\widehat{\alpha}) = 0$ and $\mu_{t+2}^* = \underline{\mu} = \lambda_1$. Regardless of previous history, if cost states *LHH* are realized in periods t, $t+1$, and $t+2$, then the prices fully adjust to the highest level in period $t+2$.

Proof. Suppose L and H are the realized cost states for periods t and $t+1$, respectively. Then by the transition equation (14), $F_{t+2}(\widehat{\alpha}) = 0$ irrespective of $F_{t+1}(\widehat{\alpha})$. By (15), $\mu_{t+2}^* = \lambda_1 = \underline{\mu}$. Therefore, if $c_{t+2} = c_H$, the prices reach the highest level, and hence are fully adjusted in period $t+2$.

Proposition 4 implies that state H is the *absorbing* state in terms of critical consumers' search behavior: regardless of the previous history, if a positive shock occurs in the current period, the search intensity will fully adjust downward in the next period. Another implication of Proposition 4 is that the prices fully adjust upward in two periods when a positive cost shock occurs and the high cost persists in the next period. More specifically, in the first period a positive shock occurs, prices only adjust upward partially because the search intensity is not fully adjusted. However, in the second period the search intensity is fully adjusted downward, leading to full upward adjustment of prices.

The key to this result is that, when a positive shock occurs, the critical consumers who search in the current period immediately learn the cost state has switched to H, and thus they stop searching in the next period. For the nonsearchers, they either observe the lower price, learn the true state H and do not search in the next period, or observe the high price, do not update their beliefs, and remain nonsearchers in the next period.

Proposition 5. Suppose a negative cost shock occurs in period $t + 1$. Then

$$F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k - \beta + \underline{\mu}\beta} < 1 \text{ and } \mu^*_{t+2} < \bar{\mu}.$$

Suppose the L state persists in the subsequent n periods after period $t + 1$. Then $F_{t+1+n}(\widehat{\alpha})$ increases in n, and converges to 1 as n goes to infinity. Hence the prices are not fully adjusted downward in period $t + 2$. Instead, the prices decrease gradually and converge to the lowest possible price as n goes to infinity.

Proof. Suppose H and L are the realized cost states for periods t and $t + 1$, respectively. According to Proposition 4, $F_{t+1}(\widehat{\alpha}) = 0$ and $\mu^*_{t+1} = \lambda_1 = \underline{\mu}$. By the transition equation (16), $F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k-\beta+\underline{\mu}\beta}$, which is clearly less than 1. If the L state persists in the subsequent n periods, by the transition equation (16), $F_{t+1+n}(\widehat{\alpha})$ will be strictly increasing in n, and eventually converges to 1 as n approaches infinity. Accordingly, the search intensity will gradually adjust upward and the prices will gradually adjust downward. The adjustment process will be completed only when n goes to infinity.

Proposition 5 implies that when a negative cost shock occurs and the low cost persists afterward, the downward price adjustment is a gradual process and takes a long time to complete. The underlying reason is that in our model, when a negative shock occurs, no critical consumer searches initially (recall that H is an absorbing state). Thus only those consumers who happen to observe the lower price learn the true state (L) and begin searching in the next period. For consumers who observe the high price, they do not learn the true state L, and remain nonsearchers in the next period. As a result, the prices do not fully adjust to the lowest level within two periods. If the L cost state persists in the subsequent n periods, more and more nonsearchers observe the lower price, learn the L state, and begin to search. Actually, the rate at which the measure of nonsearchers decreases in period t is the proportion of firms that set the lower price, $\frac{\mu^*_t\beta}{k-\beta+\mu^*_t\beta}$, by the transition equation (16).

Comparing Propositions 4 and 5, we can see the pattern of asymmetric price adjustments if the cost state persists after a shock occurs: when a positive cost shock occurs, prices fully adjust upward in two periods, whereas when a negative cost shock occurs, it takes much longer for prices to fully adjust downward. To see this asymmetry, we can evaluate the magnitude of the adjustment in the search intensity within the first two periods after a negative cost occurs. By Proposition 5, within two periods, $F(\widehat{\alpha})$ is adjusted to $\frac{\underline{\mu}\beta}{k-\beta+\underline{\mu}\beta}$. Because $\underline{\mu}$ is relatively small, this amount of adjustment is small compared to the full adjustment level (which is 1). Therefore, the adjustment of the search intensity in the first two periods is relatively small, which implies that the adjustment of prices in the first two periods is also relatively small and a significant portion of the price adjustment is completed in later periods.

Propositions 4 and 5 indicate that asymmetric price adjustments can arise if the cost shocks are persistent. Next we demonstrate that the pattern of asymmetric price adjustments does emerge on the equilibrium path of the dynamic game. Instead of following any arbitrary evolution path of the underlying state, which is a daunting task, we will focus on the *expected evolution path*, which is somewhat focused in a statistical sense. Given the persistence parameter ρ, in expectation each given state (H or L) will persist for $\frac{1}{1-\rho} \equiv N$ consecutive periods before switching to the other state ($N > 2$ because $\rho > 1/2$). We thus consider the following expected evolution path of the states: $L...LH...HL...L....$ That is, N periods of state L followed by N periods of state H, which are in turn followed by N periods of state L, and so on.

Along this expected evolution path, the lowest prices will occur in the Nth period of the L state, and the highest prices emerge in the 2nd through the Nth periods of the H state. The pattern of asymmetric price adjustment is quite striking. When a positive cost shock occurs, the prices adjust from the lowest (the Nth period of the L state) to the highest within two periods (the 2nd period of the H state), and then the prices stay at the same level until the Nth period of the H state. On the other hand, when a negative cost shock occurs, it takes $N > 2$ periods for the prices to adjust from the highest (the Nth period of the H state) to the lowest (the Nth period of the L state), before the state switches to H. During the N periods of the L state, the price decreases gradually to the lowest level. We summarize the result below.

Proposition 6. Assume $\rho > 1/2$. Along an expected evolution path, price adjustments exhibit an asymmetric pattern in equilibrium. Whereas the prices always fully adjust upward within two periods after a positive cost shock occurs, it takes a longer time for the prices to fully adjust downward after a negative cost shock occurs.

Given the random nature of the underlying state switches, the actual evolution path would not exactly follow an expected evolution path, and the duration of a given state would also be random. However, the expected evolution path can be regarded as the average overall possible evolution paths. Thus, Proposition 6 implies that the pattern of asymmetric price adjustment can emerge on the actual evolution path in a statistical sense.

The underlying reason for asymmetric adjustments lies in the asymmetric belief updating, which results from consumers' search behavior. Although searchers always learn the true cost state immediately, nonsearchers do not learn the true cost state unless they happen to observe the lower price. When there is a positive cost shock, the searchers among critical consumers immediately learn that the cost has gone up. As a result, those consumers stop searching in the next period and prices are fully adjusted upward. On the other hand, when the cost goes down, those consumers who observe the high price do not learn the true state and remain as nonsearchers, whereas only those who observe the lower price learn the true state and begin to search. As a result, the beliefs of the consumers are gradually adjusted downward as more and more nonsearchers observe the lower prices, which leads to gradual downward price adjustments.

□ **Comparative statics.** In this subsection, we study how changes in some exogenous parameters affect the pattern of price adjustments. First, we provide a measure to evaluate the degree of asymmetry in price adjustments. Denote the magnitude of upward (downward) price adjustment within two periods after a positive (negative) cost shock as MA^P (MA^N), and we define the adjustment ratio $AR = MA^N/MA^P$. AR measures the degree of asymmetry of price adjustments: the smaller AR, the smaller the magnitude of downward price adjustment within two periods relative to that of upward price adjustment within two periods, and hence the more asymmetric the price adjustments.

We start with the changes in the persistence parameter ρ. Consider two Markov processes, 1 and 2, with $\rho_2 > \rho_1 > 1/2$. That is, Markov process 2 is more persistent than process 1. In the previous analysis about the expected evolution path, we see that N increases as ρ increases. Thus we have $N_2 > N_1$. Note that $\underline{\mu} = \lambda_1$ does not depend on ρ. Hence the highest prices on the two paths are the same. However, because $N_2 > N_1$, the lowest prices under process 2 (occurring in period N_2 in state L) are lower than those under process 1 (occurring in period N_1 in state L). As a result, $MA_1^P < MA_2^P$. On the other hand, MA^N does not depend on ρ, which is evident from the equation

$$F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k - \beta + \underline{\mu}\beta} = \frac{\lambda_1\beta}{k - \beta + \lambda_1\beta}. \tag{17}$$

Thus $MA_1^N = MA_2^N$. As a result, $AR_1 > AR_2$. That is, the asymmetric price adjustments are more prominent when the cost evolution is more persistent.

So as the Markov process becomes more persistent, the magnitude of full price adjustments becomes larger. In the case of positive shocks, this larger magnitude of upward price adjustment is still completed within two periods. However, in the case of negative shocks, the downward price adjustment spreads over longer periods. This makes the price adjustment more asymmetric. The following proposition summarizes the result, which is also a testable implication.

Proposition 7. As the Markov process becomes more persistent (ρ increases), the asymmetric pattern of price adjustments becomes more prominent.

Next we consider the impact of λ_1 (the proportion of shoppers) on the pattern of price adjustment. Let $\lambda_1' < \lambda_1$. We show that the downward price adjustment in response to a negative shock is slower under λ_1' than under λ_1.

Proposition 8. As the proportion of shoppers (λ_1) decreases, the prices adjust downward more slowly when a negative cost shock occurs.

Proof. Suppose a negative shock occurs in period $t+1$, and the L state persists in the subsequent periods. Our goal is to show that the prices in any period $t+j$ ($j \geq 2$) are strictly lower under λ_1 than under λ_1'. Because prices are decreasing in the search intensity, it is sufficient to show that $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$ for all $j \geq 2$ (superscript prime is used to distinguish variables under λ_1' from those under λ_1). According to equation (17), $F_{t+2}'(\widehat{\alpha}) < F_{t+2}(\widehat{\alpha})$. Now we proceed with induction. In the first step we show that if $F_{t+j}'(\widehat{\alpha}) < F_{t+j}(\widehat{\alpha})$, then $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$ for $j \geq 2$. In the second step we prove that if $F_{t+j}'(\widehat{\alpha}) < F_{t+j}(\widehat{\alpha})$ and $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$, then $F_{t+j+1}'(\widehat{\alpha}) < F_{t+j+1}(\widehat{\alpha})$.

Step 1. By equation (14),

$$\begin{aligned}\mu_{t+j}^{*} - \mu_{t+j}^{*\prime} &= (\lambda_1 - \lambda_1') + (1 - \lambda_1 - \lambda_2)F_{t+j}(\widehat{\alpha}) - (1 - \lambda_1' - \lambda_2)F_{t+j}'(\widehat{\alpha}) \\ &= (\lambda_1 - \lambda_1')[1 - F_{t+j}'(\widehat{\alpha})] + (1 - \lambda_1 - \lambda_2)[F_{t+j}(\widehat{\alpha}) - F_{t+j}'(\widehat{\alpha})] \\ &> 0.\end{aligned}$$

Step 2. First, define

$$A_{t+j} \equiv \frac{\mu_{t+j}^{*}\beta}{k - \beta + \mu_{t+j}^{*}\beta}.$$

$A_{t+j} > A_{t+j}'$ because $\mu_{t+j}^{*} > \mu_{t+j}^{*\prime}$, and both of them are strictly between 0 and 1. Now by the transition equation (16),

$$\begin{aligned}F_{t+j+1}(\widehat{\alpha}) - F_{t+j+1}'(\widehat{\alpha}) &= [F_{t+j}(\widehat{\alpha}) - F_{t+j}'(\widehat{\alpha})] + A_{t+j}[1 - F_{t+j}(\widehat{\alpha})] - A_{t+j}'[1 - F_{t+j}(\widehat{\alpha})] \\ &= (A_{t+j} - A_{t+j}')[1 - F_{t+j}(\widehat{\alpha})] + [F_{t+j}(\widehat{\alpha}) - F_{t+j}'(\widehat{\alpha})](1 - A_{t+j}') \\ &> 0.\end{aligned}$$

Thus we obtain the desired result.

Intuitively, when the proportion of shoppers is smaller, in the case of a negative cost shock fewer consumers search initially. As a result, a smaller proportion of firms sets the lower price. This reduces the speed of learning for the critical consumers, leading to slower adjustments of the search intensity and the prices.

Although a smaller λ_1 slows down the process of downward price adjustment, the upward price adjustment is not affected: it is always completed within two periods. Thus a decrease in λ_1 makes the pattern of asymmetric price adjustments more prominent.

By a similar argument, we can show that an increase in k/β leads to a slower downward price adjustment. The intuition is that an increase in k/β, other things equal, results in fewer firms setting the lower price. This slows down the critical consumers' learning when a negative cost shock occurs, leading to a slower downward price adjustment.

4. Discussion

■ Our model is stylized, because we make a number of simplifying assumptions to facilitate our analysis. Here we discuss the restrictions of these assumptions, why we impose these assumptions, and whether they affect the general insights of the article.

First, unlike the standard search literature which considers a finite number of firms, we work with an infinite number of firms. This assumption is adopted for technical convenience. Working with a finite number of firms would involve two technical difficulties. The first difficulty is that the equilibrium of the static game (with endogenously determined search intensity) cannot be analytically derived, as shown in Tappata (2006) with more than three firms.[14] The second difficulty is that the inference about the underlying state would be too complicated in the dynamic setting. With a finite number of firms, each firm randomly chooses a price according to some distribution (usually some continuous distribution over a compact support). Thus the Bayesian updating based on different realized prices can easily get very involved. Moreover, assuming a finite number of firms adds randomness in the realized prices in each period, which further complicates the analysis of belief updating. With a continuum of firms (along with capacity constraints) in our model, we pin down a simple two-point distribution in equilibrium.[15] Moreover, the evolution of consumers' beliefs (hence equilibrium search intensity) is deterministic given the evolution of the underlying states, although individual firms might play mixed strategy in setting prices. This greatly simplifies the analysis and makes the Bayesian inferences tractable.

We believe that the general insights of our model carry over to the setting with a finite number of firms. In the static game, consumers have stronger incentives to search when they believe that the low-cost state is more likely, because in a low-cost state prices are more dispersed due to competition among firms; in the dynamic setting, asymmetry in learning continues to be present. Searchers observe all the prices set by the firms, whereas each nonsearcher only observes one price and, consequently, the searchers learn the true state more quickly than nonsearchers. All these features do not seem to be restricted to the setting with an infinite number of firms. Thus our assumption of an infinite number of firms is mainly for technical convenience, which helps simplify consumers' Bayesian inferences and enables our equilibrium analysis to be tractable.[16]

Note that another crucial assumption for our equilibrium analysis is the capacity constraint, without which price dispersion simply does not arise in our equilibrium.[17] Although capacity constraints are prevalent in many retail markets, our specific assumption that all firms have the same capacity may appear restrictive. Again, we maintain this assumption mainly for technical convenience, which should not affect the robustness of our main results. To see this, we consider an alternative setting in which a proportion γ of firms each has capacity k_2, and the rest of the firms each have capacity k_1, where $k_2 > k_1 > \beta$. We assume that $\gamma k_2 < \beta$, so that low-capacity firms are viable. In this setting, the static equilibrium with fixed search intensity μ is still characterized by a two-point distribution.[18] Depending on parameter values, we may have two types of equilibria. In the first-type equilibrium, firms with the bigger capacity k_2 charge the low price $\underline{p}$ with probability 1, and firms with the smaller capacity k_1 mix between $\underline{p}$ and the choke price $\bar{v}$. Let $\eta_1(\underline{p})(\eta_2(\underline{p}))$ denote the probability with which a firm with capacity $\bar{k}_1(k_2)$ charges $\underline{p}$. Then the equilibrium is determined by the following two equations that are similar to (1) and (2):

$$\pi_1(v) = \pi_1(\underline{p}) \Leftrightarrow (1-\mu)\beta(v-c) = k_1(\underline{p}-c) \tag{18}$$

[14] The main reason is that the expected gain of search cannot be integrated out explicitly.

[15] The two-point price distribution derived in our model is somewhat special; nevertheless, we believe that it captures the essence of real-world product market price dispersions while enabling our analysis to be tractable.

[16] Note that our approach is not without a cost—an undesirable feature of our equilibrium is that it lacks an appropriate *continuity* (in the number of firms); in fact, our current equilibrium construction does not seem to work with any finite number of firms.

[17] Recall that the capacity constraint keeps the profit margin of $\underline{p}$ firms positive by restricting the competition among (infinitely many) $\underline{p}$ firms.

[18] The arguments are similar to Lemmas 1–4. Firms charging the lowest price have sales equal to their capacities. Searchers are not rationed at the lowest price.

$$\mu\beta + (1-\mu)\beta[\gamma + (1-\gamma)\eta_1(\underline{p})] = k_1(1-\gamma)\eta_1(\underline{p}) + k_2\gamma. \tag{19}$$

Equation (18) says that a firm with capacity k_1 is indifferent between charging v and charging $\underline{p}$, and equation (19) says that the total demand for the firms charging $\underline{p}$ equals the total capacity of those firms. Note that firms with capacity k_2 have no incentive to charge v, because $\pi_2(\underline{p}) = k_2(\underline{p} - c) > \pi_1(\underline{p}) = \pi_1(v) = \pi_2(v)$. This is the unique equilibrium if $\mu\beta + (1-\mu)\beta\gamma \geq k_2\gamma$ (the search intensity is big enough).

On the other hand, if $\mu\beta + (1-\mu)\beta\gamma < k_2\gamma$, then the following (second-type) equilibrium is unique: firms with capacity k_2 mix between v and $\underline{p}$ with the probability of charging $\underline{p}$ being $\eta_2(\underline{p})$, and firms with capacity k_1 charge v with probability 1. Similarly to equations (18) and (19), the equations characterizing the equilibrium are as follows:

$$\pi_2(v) = \pi_2(\underline{p}) \Leftrightarrow (1-\mu)\beta(v-c) = k_2(\underline{p} - c)$$
$$\mu\beta + (1-\mu)\beta\gamma\eta_2(\underline{p}) = k_2\gamma\eta_2(\underline{p}).$$

Note that firms with capacity k_1 have no incentive to charge v because firms with capacity k_2 are indifferent between charging v and charging $\underline{p}$.

The rest of the analysis follows qualitatively as in our base model. With the equilibrium prices being distributed according to a two-point distribution, we can endogenously determine the search intensity given the distribution of beliefs among critical consumers. The heterogeneity among firms does not qualitatively affect consumers' learning asymmetry, although the dynamics of price adjustment will be slightly modified.

We follow Varian (1980) to assume that searchers observe *all* the realized prices. Given that we are working with a continuum of firms, this assumption effectively implies that searchers can observe infinitely many prices. This "infinite observability" is somewhat strong, but again we maintain this assumption for tractability purposes. To see this, we consider an alternative setting in which searchers are only able to observe a finite number (n) of realized prices, as in Burdett and Judd (1983). First of all, the equilibrium distribution of prices given μ, the fixed search intensity, becomes nonatomic. Following similar derivations in Burdett and Judd, the equilibrium price distribution ($G(\cdot \mid c, \mu)$) is determined by the following condition:

$$(1-\mu)\beta(v-c) = (p-c)\left[(1-\mu)\beta + \mu\beta n(1 - G(p \mid c, \mu))^{n-1}\right],$$

which basically requires that each firm be indifferent between charging the choke price and charging any other price over the support of randomization. Solving, we have the equilibrium price distribution

$$G(p \mid c, \mu) = 1 - \left(\frac{1-\mu}{\mu n}\frac{v-p}{p-c}\right)^{1/n-1}, \quad p \in [\underline{p}, v],$$

where the lower bound of the equilibrium support is determined by $G(\underline{p} \mid c, \mu) = 0$.

It turns out that given this nonatomic equilibrium price distribution, the static equilibrium with endogenously determined search intensity cannot be derived analytically, because the expected gain of search cannot be explicitly integrated out.[19] By assuming that searchers observe all the prices (so that they always receive the lowest price), we obtain a two-point price distribution and effectively get around this technical difficulty with equilibrium characterization. Although a full analysis is not attempted, we believe that our general insights of asymmetric price adjustment carry over to the setting under the presumably more reasonable assumption of finite observability. Given that searchers observe n realized prices and nonsearchers only observe one realized price, searchers learn the true cost state quicker than nonsearchers. This learning asymmetry is the main driving force for the asymmetric price adjustment: when a positive cost shock occurs, searchers learn it quickly and stop searching, which leads to a rather quick upward price adjustment; when a negative shock occurs, nonsearchers learn it slowly, which leads to a slow downward price

[19] This can be verified by following similar calculations as in Tappata (2006).

adjustment. Obviously, the asymmetry in price adjustment will be most prominent when the searchers can observe all the prices. In this sense, our assumption can be regarded as the limiting case when n goes to infinity.

Finally, in the dynamic game, we make the nonoverlapping assumption (11) such that the equilibrium ranges of $\underline{p}_H$ and $\underline{p}_L$ do not overlap. As a result, whenever a nonsearcher observes the lower price, she learns the true cost state immediately. In a more general model, we should allow for the case that $\underline{p}_H$ and $\underline{p}_L$ may overlap in equilibrium. Under such a setting, we believe that the general insight of the article still holds. To see this, note that searchers can always infer the true cost state immediately. They have two pieces of information, the lower price $\underline{p}$ and the proportion of firms charging the lower price $\eta(\underline{p})$. Thus, from two equations (3) and (4), the searchers can correctly infer the two unknowns, μ and c. However, the belief updating for nonsearchers would be much more involved. If a nonsearcher observes a lower price which lies in the overlapping range, she cannot immediately infer the true state. Although it is difficult to pin down the exact belief-updating process in this case,[20] it is clear that nonsearchers' belief updating is slower than that of the searchers. Given this, asymmetric price adjustment will emerge as well.

5. Conclusion

■ In this article, we build a simple search model with learning to demonstrate how asymmetric price adjustments can arise as firms' optimal responses to cost shocks. Although the upward price adjustment is always completed within two periods after a positive cost shock occurs, the downward price adjustment takes much longer to complete when a negative shock occurs.

The underlying reason for asymmetric price adjustments is that searchers and nonsearchers have different belief-updating processes. Because searchers observe the whole spectrum of price distribution whereas nonsearchers only observe one single price, searchers learn the true cost state a lot quicker than nonsearchers do. This learning asymmetry naturally leads to asymmetric price adjustments. When a positive cost shock occurs, searchers quickly learn the true state and stop searching. Thus the quick downward adjustment of search intensity leads to the quick upward price adjustment. On the other hand, when a negative cost shock occurs, it takes a much longer period of time for nonsearchers to learn the true state and start searching. This slow upward adjustment of search intensity leads to slow downward price adjustment.

Thus, our article provides an explanation for the widespread phenomenon of asymmetric price adjustments. Although our model is simple, we believe that it captures the essence of real-world product markets with imperfectly informed consumers. Our model also predicts that asymmetric price adjustments are more prominent in markets where the cost shocks are more persistent or where more critical consumers are present.

References

BACON, R.W. "Rockets AND Feathers: The Asymmetric Speed of Adjustment of UK Retail Gasoline Prices to Cost Changes." *Energy Economics,* Vol. 13 (1991), pp. 211–218.

BENABOU, R. AND GERTNER, R. "Search with Learning from Prices: Does Increased Inflationary Uncertainty Lead to Higher Markups?" *Review of Economic Studies,* Vol. 60 (1993), pp. 69–93.

BORENSTEIN, S., CAMERON, A.C., AND GILBERT, R. "Do Gasoline Prices Respond Asymmetrically to Crude Oil Price Changes?" *Quarterly Journal of Economics,* Vol. 112 (1997), pp. 305–339.

BOYD, M.S. AND BRORSEN, B.W. "Price Asymmetry in the U.S. Pork Marketing Channel." *North Central Journal of Agricultural Economics,* Vol. 10 (1988), pp. 103–110.

BRAVERMAN, A. "Consumer Search and Alternative Market Equilibria." *Review of Economic Studies,* Vol. 47 (1980), pp. 487–502.

BURDETT, K. AND JUDD, K.L. "Equilibrium Price Dispersion." *Econometrica,* Vol. 51 (1983), pp. 955–970.

[20] We need to derive the distribution of the lower price in the dynamic game (which is a daunting job), and then apply Bayesian rule to pin down the belief-updating process.

CABRAL, L. AND FISHMAN, A. "A Theory of Asymmetric Price Adjustment." Department of Economics, Stem School of Business, New York University, 2006.

DANA, J.D. "Learning in an Equilibrium Search Model." *International Economic Review,* Vol. 35 (1994), pp. 745–771.

DELTAS, G. "Retail Gasoline Price Dynamics and Local Market Power." Department of Economics, University of Illinois, Urbana-Champaign, 2004.

DUFFY-DENO, K.T. "Retail Price Asymmetries in Local Gasoline Markets." *Energy Economics,* Vol. 18 (1996), pp. 81–92.

ECKERT, A. "Retail Price Cycles and Response Asymmetry." *Canadian Journal of Economics,* Vol. 35 (2002), pp. 52–77.

FISHMAN, A. "Search with Learning and Price Adjustment Dynamics." *Quarterly Journal of Economics,* Vol. 111 (1996), pp. 253–268.

GOODWIN, B. AND HARPER, D. "Price Transmission, Threshold Behavior, and Asymmetric Adjustment in the U.S. Pork Sector." *Journal of Agricultural and Applied Economics,* Vol. 32 (2000), pp. 543–553.

———, AND HOLT, M.T. "Price Transmission and Asymmetric Adjustment in the U.S. Beef Sector." *American Journal of Agricultural Economics,* Vol. 81 (1999), pp. 630–637.

HANNAN, T.H. AND BERGER, A.N. "The Rigidity of Prices: Evidence from the Banking Industry." *American Economic Review,* Vol. 84 (1991), pp. 938–945.

KARRENBROCK, J.D. "The Behavior of Retail Gasoline Prices: Symmetric or Not?" *Federal Reserve Bank of St. Louis Review*, July/August (1991), pp. 19–29.

LEWIS, M. "Asymmetric Price Adjustment and Consumer Search: An Examination of the Retail Gasoline Market." Department of Economics, The Ohio State University, 2005.

NEUMARK, D. AND SHARPE, S.A. "Market Structure and the Nature of Price Rigidity: Evidence from the Market for Consumer Deposits." *Quarterly Journal of Economics,* Vol. 107 (1992), pp. 657–680.

O'BRIEN, J. "Estimating the Value and Interest Rate Risk of Interest-Bearing Transactions Deposits." Finance and Economics Discussion Paper no. 46, Board of Governors of the Federal Reserve System, 2000.

PELTZMAN, S. "Prices Rise Faster Than They Fall." *Journal of Political Economy,* Vol. 108 (2000), pp. 466–502.

PICK, D.H., KARRENBROCK, J., AND CARMEN, H.F. "Price Asymmetry and Marketing Margin Behavior: An Example for California-Arizona Citrus." *Agribusiness,* Vol. 6 (1991), pp. 75–84.

ROB, R. "Equilibrium Price Distributions." *Review of Economic Studies,* Vol. 52 (1985), pp. 487–504.

ROTEMBERG, J. AND SALONER, G. "A Supergame-Theoretical Model of Price Wars during Booms." *American Economic Review,* Vol. 76 (1986), pp. 390–407.

SALOP, S. AND STIGLITZ, J. "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion." *Review of Economic Studies,* Vol. 44 (1977), pp. 493–510.

STAHL, D.O. "Oligopolistic Pricing with Sequential Consumer Search." *American Economic Review,* Vol. 79 (1989), pp. 700–712.

TAPPATA, M. "Rockets and Feathers: Understanding Asymmetric Pricing." Strategy and Business Economics Division, Sauder School of Business, University of British Columbia. 2006.

VARIAN, H.R. "A Model of Sales." *American Economic Review,* Vol. 70 (1980), pp. 651–659.

WARD, R.W. "Asymmetry in Retail, Wholesale and Shipping Point Pricing for Fresh Vegetables." *American Agricultural Economics Association,* Vol. 14 (1982), pp. 205–212.