QUALITY DISCLOSURE AND COMPETITION*

DAN LEVIN†
JAMES PECK‡
LIXIN YE§

We analyze costly quality disclosure with horizontally differentiated products under duopoly and a cartel, and characterize the effect of competition on disclosure and welfare. We show that expected disclosure is higher under a cartel than under duopoly, and the welfare comparison depends on the level of disclosure cost: when the disclosure cost is low, welfare is higher under a cartel than duopoly, but when the disclosure cost is high, welfare is higher under duopoly. In either market structure, disclosure is excessive in terms of total surplus, but insufficient in terms of consumer surplus.

I INTRODUCTION

A free and efficient flow of information is important for modern, information driven economies. Although prices are the main carriers of information critical for decision making, important information is revealed to consumers directly by firms, either voluntarily or as mandated by law. For example, drug manufacturers must incur substantial costs in order to certify the safety and efficacy of new

*We are very grateful to the editor and three anonymous referees for their insightful comments. All remaining errors are our own.

†Authors' Affiliation: The Ohio State University, Department of Economics, 1945 N. High St., Columbus, OH 43210-1172, USA.
email: levin.36@osu.edu.

‡The Ohio State University, Department of Economics, 1945 N. High St., Columbus, OH 43210-1172, USA.
email: peck.33@osu.edu

§The Ohio State University, Department of Economics, 1945 N. High St., Columbus, OH 43210-1172, USA.
email: lixinye@econ.ohio-state.edu

drugs. A substantial portion of these costs can be attributed to the cost of *disclosing* the quality, as opposed to the cost of *learning* the quality.[1] In other industries, disclosure is not regulated, yet firms choose to incur expenses in order to disclose quality. What determines firms' disclosure behavior? What is the motive for a firm to reveal its private information? In a market with inefficient disclosure, how can a regulatory policy correct the inefficiency? We are motivated by these questions, and in particular, by how the answers depend on the market structure in the relevant industries.

As far as we know, we are the first to address these questions in a model with *all* of the following features: (i) quality is privately observed by firms, (ii) after observing quality, firms can credibly disclose it by incurring a positive cost, (iii) products are differentiated in terms of quality and a non-quality characteristic like flavor, and (iv) a monopoly cartel and duopoly are both considered. In particular, we consider a Hotelling model of price competition with differentiated products. Consumers are located along the unit interval, interpreted to be the most preferred product characteristic, and the two firms are located at either endpoint. Each firm's product is also defined along a second dimension, interpreted to be product quality. Product quality is privately observed by the firm, and the two qualities are independent. In the symmetric duopoly equilibrium we characterize, each firm chooses to disclose if and only if its quality is above a certain threshold, q^{*D}. Prices depend on the perceived or observed difference in the firms' qualities.

For the analogous cartel model, two firms within the cartel share the information about both qualities and can perfectly coordinate on the disclosure decisions. Our cartel can be interpreted as a multi-product monopoly (with product characteristics fixed), which could also arise if products of two existing brands were to merge. While under duopoly each firm cares about its own profit, a cartel maximizes the joint profit of the two products. Thus, our cartel model can be interpreted as any of several ownership structures (cartel, monopoly, collusion, joint-venture), under the restriction that product characteristics are not a choice variable. Allowing a location choice would affect our results because a cartel/monopoly might choose different locations than the two duopolists would

[1] A firm sometimes learns that a drug approved to treat one condition is effective in treating a completely different condition. The firm might have evidence that the drug is of high quality, but it cannot advertise this effectiveness without undertaking costly trials. When the firm decides to undertake the trials, the cost is a disclosure cost and not an information acquisition cost.

choose.

The cartel must decide which of the two qualities to disclose. We find that, if both qualities are below a threshold, q^{*C}, then nothing is disclosed. If the higher quality is above q^{*C} and the lower quality is below a threshold (that depends on the higher quality), then only the higher quality is disclosed. Otherwise, both qualities are disclosed. Thus the equilibrium disclosure in the cartel case exhibits some nontrivial coordination between two quality realizations. Again, prices depend on the two perceived (or disclosed) qualities.[2]

In Section 5, based on the equilibria we characterize, we compute expected levels of disclosure and welfare for the duopoly model and the cartel model. One might think that there would be more disclosure under duopoly, because a duopolist ignores the losses that disclosing imposes on its rival. Surprisingly, we find that the expected amount of disclosure is always higher under a cartel than under duopoly. The intuition is that, if $q^{*D} = q^{*C}$ were to hold, a cartel with the threshold quality appropriates all of the benefits of disclosure, and raises both of its prices. When a duopolist with the threshold quality discloses, it raises its price by less than the average cartel price increase, and its market share does not go up by enough to compensate. Since a cartel with the threshold quality is indifferent between disclosing and not disclosing, a duopolist with the same quality strictly prefers not to disclose, implying $q^{*D} > q^{*C}$.

As to the welfare comparison, we show that for small values of the disclosure cost, δ, welfare is higher under a cartel, but for large values of δ, welfare is higher under duopoly. Two sources of welfare loss account for this result: excessive disclosure cost and misallocation. The welfare loss, due to excessive disclosure costs being incurred, is greater for a cartel than for duopoly, but relatively unimportant when δ is either small (disclosure is not very costly per unit) or high (disclosure is rare). A duopolist will take advantage of a higher quality perception partly by raising its price and partly by expanding its market share. The expansion of market share turns out to be socially excessive, so the allocation of consumers to products is inefficient. Because a cartel internalizes the impact of an expanding market share of one product on the declining market share of the other product, this misallocation is greater for duopoly than for a cartel.[3] For low δ, the welfare loss due

[2] All of our main results go through for the model in which each firm in the cartel is constrained to choose a disclosure cutoff that does not depend on the perceived product quality of the other firm. In particular, $q^{*C} < q^{*D}$ holds, expected disclosure is higher under a cartel, and the welfare comparison favors a cartel for low δ and duopoly for high δ.

[3] All consumers are served in equilibrium, so we abstract from the effect of imperfect competition on total output.

to misallocation dominates, and a cartel yields higher welfare than duopoly. For moderate δ, the welfare loss due to excessive disclosure costs being incurred dominates, and duopoly yields higher welfare. For large δ, there is no disclosure and welfare is the same under both cartel and duopoly.

In Section 6, we introduce the benchmark of a social planner, with the ability to control disclosure cutoffs but not prices. This allows us to compute the socially optimal levels of disclosure for the duopoly model and the cartel model. In both models, the equilibrium levels of disclosure are excessive. Although there is excessive disclosure from the perspective of society as a whole, we find that consumer surplus is higher when more disclosure is mandated. Based on the political economy of consumer interests versus societal interests, we can reconcile the typical result in the literature, excessive disclosure, with the 'man in the street' view that mandated disclosure is a good thing.

In Section 7, we first discuss a possible modification to the model that allows for equilibrium under-disclosure. By supposing that only a fraction of the disclosure cost represents a true social cost, the welfare calculations change without affecting the equilibrium, though equilibrium disclosure can be insufficient. Alternatively, the cost of disclosing a low quality might be the loss of revenue from consumers who do not update their priors unless a disclosure announcement is made (see Hirshleifer, Lim and Teoh [2002]). Under this behavioral approach, we would also have equilibrium under-disclosure, but arguments for government intervention would rest on the assumption of bounded rationality. Finally, to better understand the source of excessive disclosure in our analysis, we consider the case in which firms can commit to disclosure cutoffs (before qualities are realized). Interestingly, we find that with commitment, equilibrium disclosure will be insufficient, rather than excessive in both market structures. This finding shows that our excessive disclosure results depend crucially on the inability of a firm to commit to its disclosure policy before observing its quality.

II LITERATURE REVIEW

An important result in the early literature is that, if disclosure is costlessly credible, a privately informed seller will voluntarily disclose all information. This is the well known unraveling result (see, for example, Grossman and Hart, 1980, or Milgrom and Roberts, 1986, in the monopoly context, and Okuno-Fujiwara, Postlewaite, and Suzamura, 1990, in the oligopoly context). In such environments,

In other models, this can be an important source of inefficiency, especially for a cartel.

mandatory disclosure rules are unnecessary.[4] However, we often do not observe full disclosure in practice. To reconcile the gap between the theoretical predictions and this empirical observation, other studies utilize one of two approaches.

The first approach assumes that disclosing is costly. Viscusi [1978] and Jovanovic [1982] show that if disclosure is costly, sellers will voluntarily disclose only if their quality exceeds some threshold. Introducing a disclosing cost thus leads some sellers to choose not to disclose. However, the analysis does not really justify the motive for a mandatory disclosure rule. In their models, consumers are equally well off with and without disclosure, since the price they pay equals their expected valuation conditional on their information. Sellers may be even worse off with mandatory disclosure, because it eliminates the option to withhold information and save the disclosure cost. Jovanovic shows that, in equilibrium, there is actually too much disclosure. A small subsidy for sales in which quality is not disclosed improves welfare.

The second approach employs 'behavioral' models that assume that not all consumers are fully rational. Hirshleifer et al. [2002] assume that there are two types of consumers: fully rational consumers who could correctly infer from firms' disclosure the relevant quality, and boundedly rational consumers who naively neglect to draw inferences from the fact that a firm does not disclose quality, believing instead that the distribution of quality is the prior distribution. The presence of boundedly rational consumers introduces at least some incentive for a seller not to disclose information about quality, even if it is costless. Qualitatively speaking, these behavioral models are similar to the disclosure-cost models. In both types of models, unraveling of the decision not to disclose is only partial. For a firm with sufficiently low quality, either separating oneself from the pool of lowest-quality firms is not worth the cost of disclosure, or separating oneself from the pool of lowest-quality firms is not worth the foregone earnings from naive consumers.

Fishman and Hagerty [2003] adopt a monopoly model with two types of consumers: those who fully understand disclosed information, and those who observe whether information was disclosed

[4] Matthews and Postlewaite [1985] turn common intuition on its head, by considering a model in which the firm chooses whether to learn the quality of its product, and then chooses whether to disclose the quality. Both learning and disclosing are costless. Since the firm cannot credibly argue that it did not learn its quality, quality is learned and disclosed. However, a mandatory disclosure law might give an incentive for the firm not to observe quality, because now the firm could credibly argue that it did not learn its quality, because then it would have been disclosed. Thus, mandatory disclosure laws can lead to less disclosure.

but not the content of what was disclosed. These uninformed consumers are fully rational, and draw inferences about quality from the price and from whether or not the firm discloses. Fishman and Hagerty construct equilibria in which both seller types do not disclose, even though disclosure is costless.[5]

There has been a recent literature addressing the effects of competition on disclosure. However, this literature does not adequately consider what is arguably the most important framework, namely, a framework with 'universal private information,' in which products are differentiated in terms of quality and a non-quality characteristic.[6] In Dye and Sridhar [1995], increasing competition may increase disclosure, which is opposite to our result. Rather than measuring the quality of the product sold by the firm, information is about the expected profitability or cash flows of the firm, and firm managers seek to maximize the firm's stock price. Another distinction is that traders are not sure whether firms observe a quality signal in Dye and Sridhar [1995]. Stivers [2004] adapts Dye and Sridhar's model to oligopolistic competition with vertically differentiated products. He adopts a reduced-form approach, and the structural example presented is inconsistent with our setup of heterogeneous consumer preferences over products. Stivers [2004] finds that increasing competition (either by increasing the number of firms or increasing the intensity of competition among firms) strengthens quality disclosure.

Board [2003] examines a duopoly model with consumers that have heterogeneous preferences for quality (vertically differentiated products), and where disclosing quality is costless. He assumes that both firms know the quality of both products, but consumers cannot observe quality unless it is disclosed. Even though disclosure is costless, full unraveling does not occur, so competition and disclosure can be inversely related. The higher quality is always disclosed, and the lower quality is disclosed if it is neither too high nor too low. The intuition is that, if the two qualities are almost the same, then we are nearly in the situation of pure Bertrand competition, so the lower quality firm is better off pooling with its low quality types. This yields most of the market profits to the higher quality firm, but profitably serves consumers with relatively low demand for

[5] The recent literature on competition and disclosure (see below) also contains examples where some types do not make costless disclosures.

[6] Daughety and Reinganum [2007] define universal private information to mean that firms privately observe the quality of their products, and consumers privately observe their preferences.

quality. Hotz and Xiao [2005] allow for both vertically and horizontally differentiated products in a duopoly model with costless disclosure. Firms observe the qualities of both products, which are perfectly negatively correlated. They avoid full unraveling, again because disclosure leads to fiercer price competition. Cheong and Kim [2004] examine the effect of competition on disclosure, but assume that consumers are homogeneous. As a result, only firms that disclose receive customers in equilibrium.[7] Furthermore, the comparison between monopoly and duopoly essentially compares two different aggregate technologies, because the highest quality product under duopoly is likely to be higher than the quality of the monopoly product.

In our model, quality is a firm's private information unless disclosed, consumers are heterogeneous in their demand for product characteristics other than quality, and disclosure is costly. Our framework also allows us to identify the role of commitment in affecting equilibrium disclosure. A complementary approach is adopted by Daughety and Reinganum [2005, 2007]. As in our model, they make the natural assumption that quality is a firm's private information. But unlike in our analysis, they model the disclosure choice as occurring before a firm learns its quality. This commitment dramatically changes the impact of competition on disclosure.[8]

III THE DUOPOLY MODEL

There are two firms producing a differentiated product, represented by firm 0 and firm 1 located at either endpoint of the unit interval. Products 0 and 1 differ on two dimensions: the 'taste' dimension and the quality dimension. Consumers all have the same preference for high quality, but different consumers have different preferences along the taste dimension. We model the taste dimension as the location of a consumer within the unit interval, representing the ideal product characteristic for that consumer. The distance from a consumer's location to the product location is the loss in utility from consuming a product that is less than ideal. Thus, consumers located near 0 have a strong preference for product 0 over product 1, consumers located near 1 have a strong preference for product 1 over 0, and a consumer located at 1/2 is indifferent between the two products (holding quality constant). A consumer is thus characterized by the distance, x, from the location of firm

[7] In a supplementary appendix, they briefly consider consumer heterogeneity. However, heterogeneity is over demand for quality, and not the products themselves (vertical but not horizontal differentiation).

[8] Because theirs is a multiple period model, pricing in period 1 can signal quality in equilibrium.

0 (x can be regarded as the 'type' of the consumer). We assume that consumers are uniformly distributed along the unit interval.

For $i = 0, 1$, the quality of the good produced by firm i is $\psi + q_i$, where ψ is a common value component exogenously given in our model and q_i's are independent draws from the uniform distribution over $[0, 1]$. A consumer purchases either 0 or 1 unit of output, where the utility of not purchasing is normalized to zero. Given q_0 and q_1, the values of product 0 and 1 to a type-x consumer are given by $\psi + q_0 - x$ and $\psi + q_1 - (1 - x)$, respectively. Thus, given prices p_0 and p_1 charged by firms 0 and 1, respectively, a type-x consumer's utilities obtained from purchasing one unit of products 0 and 1, are given by $\psi + q_0 - p_0 - x$, and $\psi + q_1 - p_1 - (1 - x)$, respectively.

For simplicity, we assume $\psi > \frac{3}{2}$, which guarantees that in equilibrium all consumers will be served in the market. Since the value of ψ is common knowledge to all parties in the model, henceforth we simply refer to q_i as the quality of product i, $i = 0, 1$. The consumer will either not purchase, purchase one unit of product 0, or purchase one unit of product 1, depending on which decision induces the highest expected utility conditional on the consumer's available information.

The time line of the duopoly game is as follows. First, each firm privately observes its quality q_i, and decides whether or not to disclose it. Thus, each firm's disclosure strategy can be represented by a function $D_i : [0, 1] \to \{0, 1\}$, mapping quality to a disclosure choice, where $D_i = 0$ means 'do not disclose' and $D_i = 1$ means 'disclose.' We assume that certifying a false quality is impossible. The cost of disclosing is denoted by δ, and the marginal cost of production is normalized to zero. After the disclosure choices have been made and observed by both firms, the firms simultaneously choose prices.[9] Next, consumers observe the disclosure and pricing choices of the two firms, and make their purchasing decisions. Given that consumers are risk neutral, consumer behavior depends on the expected qualities of the two goods, conditional on the firms' disclosure and pricing choices.

[9] If prices were set simultaneously with the disclosure decision, then a duopolist would have to set prices without knowing what consumers believe about the other product's quality. Our time line eliminates this discrepancy between duopoly and a cartel, allowing a cleaner comparison between duopoly and a cartel. Our main results are robust to specifying a simultaneous price and disclosure choice. Namely, a cartel is more likely to have at least one product disclosed than duopoly, and the expected total surplus is higher under a cartel than duopoly for small δ, higher under duopoly for intermediate δ. However, the expected number of disclosed products could be higher under duopoly for some small δ.

We will refer to these conditional expected qualities as *perceived quality*, denoted by $\widetilde{q}_0$ and $\widetilde{q}_1$.[10]

Our solution concept is Perfect Bayesian Equilibrium (PBE), which consists of the equilibrium strategies (firms' disclosure and pricing strategies and the consumers' purchasing strategies), and the consumers' equilibrium beliefs about product qualities. Because quality does not affect production cost, the profit maximizing prices, (p_0, p_1), do not depend on the value of any undisclosed quality. There is no signalling role for prices in our model. Thus, we feel justified in restricting attention to PBE in which consumers' beliefs about undisclosed qualities do not depend on either price.

Definition 1: A *duopoly equilibrium* is a symmetric PBE, in which there is a quality threshold, q^{*D}, such that (1) firm i discloses its quality q_i if and only if $q_i > q^{*D}$; and (2) if firm i does not disclose its quality, beliefs about q_i are independent of the firm's pricing choice.

It is easily seen that the firms' profits only depend on the prices and the perceived qualities, instead of the true qualities. Since perceived qualities depend on the disclosure choices and not the prices, each firm's pricing choice can be represented as a function of the two perceived qualities. In equilibrium, there will be a cutoff type of consumer, x^*, where consumers with types $x < x^*$ purchase from firm 0, consumers with types $x > x^*$ purchase from firm 1, and consumers with type $x = x^*$ are indifferent between purchasing from firm 0 and firm 1. Given such cutoff x^*, we simply say that the market share for product 0 is x^*.

Given the prices (p_0, p_1), the perceived qualities $(\tilde{q}_0, \tilde{q}_1)$, and assuming for the moment that all consumers prefer purchasing from one of the firms to not purchasing, x^* is determined by the following equation:

$$\psi + \tilde{q}_0 - p_0 - x^* = \psi + \tilde{q}_1 - p_1 - (1 - x^*),$$

which gives

$$x^* = \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 - \tilde{q}_1 + p_1 - p_0). \tag{1}$$

[10] Obviously, if the quality of product i is disclosed, we have $\widetilde{q}_i = q_i$.

The firms' profits are given by:

$$\begin{aligned}\pi_0(p_0,p_1) &= p_0 x^* = p_0\left[\frac{1}{2}+\frac{1}{2}(\tilde{q}_0-\tilde{q}_1+p_1-p_0)\right] \quad \text{and} \\ \pi_1(p_0,p_1) &= p_1(1-x^*) = p_1\left[\frac{1}{2}-\frac{1}{2}(\tilde{q}_0-\tilde{q}_1+p_1-p_0)\right].\end{aligned}$$

Sequential rationality requires the equilibrium prices to be

$$p_0 = 1+\frac{1}{3}(\tilde{q}_0-\tilde{q}_1) \text{ and} \tag{2}$$

$$p_1 = 1+\frac{1}{3}(\tilde{q}_1-\tilde{q}_0). \tag{3}$$

Substituting the prices from (2) and (3) into the expressions for market share and profits, we can express market share and profits as a function of perceived qualities, given by

$$x^{*D}(\tilde{q}_0,\tilde{q}_1) = \frac{1}{2}+\frac{1}{6}(\tilde{q}_0-\tilde{q}_1), \tag{4}$$

$$\pi_0(\tilde{q}_0,\tilde{q}_1) = \frac{1}{18}(3+\tilde{q}_0-\tilde{q}_1)^2, \tag{5}$$

$$\pi_1(\tilde{q}_0,\tilde{q}_1) = \frac{1}{18}(3+\tilde{q}_1-\tilde{q}_0)^2.$$

From (1)-(3) and our assumption, $\psi > \frac{3}{2}$, it is straightforward to demonstrate that all consumers are willing to purchase from one of the firms, even if the quality is known to be zero.[11]

Proposition 1 *There is a unique duopoly equilibrium. The disclosure threshold q^{*D}, is given by*

$$q^{*D} = \begin{cases} -\frac{5}{3}+\frac{1}{3}\sqrt{25+216\delta} & \text{if} \quad \delta\in[0,13/72] \\ 1 & \text{if} \quad \delta > 13/72 \end{cases} \tag{6}$$

and consumers believe that an undisclosed quality is uniformly distributed over $[0,q^{*D}]$, so that the perceived quality is $\tilde{q}_i = q^{*D}/2$. Prices are given by (2) and (3) and consumer behavior is characterized by the market share expression, (4).

Proof. Because disclosing a quality of zero is costly and can never raise a firm's perceived quality, not disclosing must be on the equilibrium path, so the specified consumer beliefs follow immediately

[11] With $\psi < 3/2$, then for some δ and some quality realizations, some consumers are not served in the duopoly equilibrium. This is an interesting possibility requiring separate analysis. To keep the paper to a reasonable length, we leave this analysis to future work.

from Bayes' rule. Suppose firm 1 follows a disclosure strategy characterized by a cutoff quality q^*. If firm 0 discloses its quality q_0, its expected profit from the following duopoly competition is given by

$$q^* \left[\frac{1}{18}(3 + q_0 - q^*/2)^2 \right] + \int_{q^*}^{1} \frac{1}{18}(3 + q_0 - q)^2 \, dq - \delta. \tag{7}$$

If firm 0 does not disclose its quality, its expected profit is given by

$$q^* \cdot \frac{1}{2} + \int_{q^*}^{1} \frac{1}{18}(3 + q^*/2 - q)^2 \, dq. \tag{8}$$

Note that given q^*, profit from disclosing in expression (7) is strictly increasing in q_0, and profit from not disclosing in expression (8) is independent of q_0. Therefore, for q^* to characterize an equilibrium threshold with $q^* < 1$, a necessary and sufficient condition is that expressions (7) and (8) be equal at $q_0 = q^*$. For $\delta \leq \frac{13}{72}$, the unique solution is $q^{*D} = -\frac{5}{3} + \frac{1}{3}\sqrt{25 + 216\delta}$. There cannot be an equilibrium with $q^{*D} = 1$, because firm 0 would receive higher profits by disclosing when $q_0 = 1$ occurs.

For $\delta > \frac{13}{72}$, there cannot be an equilibrium with an interior threshold. However, there is an equilibrium with no disclosure, characterized by $q^{*D} = 1$. Each firm has a perceived quality of $\frac{1}{2}$, chooses a price of 1, and receives expected profits of $\frac{1}{2}$. The disclosure cost is high enough so that a deviation to disclose is not profitable, even with the highest possible quality. If firm i discloses quality, $q_i = 1$, the resulting price competition yields profits of $\frac{49}{72} - \delta$. Since we have $\delta > \frac{13}{72}$, this deviation is not profitable. All consumers are willing to purchase, so (6) characterizes a duopoly equilibrium. □

IV THE CARTEL MODEL

In order to evaluate how the degree of competition affects disclosure and welfare, we now consider a cartel model that is otherwise identical to the model of Section 3. The locations and utility functions of consumers is the same, the location of products is the same, and the joint distribution of qualities is the same. The only difference is that now two firms in the cartel share the information about both qualities, and can perfectly coordinate on quality disclosure to maximize the joint profit.

Based on the realizations of both qualities, the cartel chooses to disclose either none, one, or both of the qualities. Thus the disclosure strategy $D : [0,1] \times [0,1] \to \{(0,0),(1,0),(0,1),(1,1)\}$ maps a pair of qualities to a disclosure choice. After the disclosure decision, the cartel sets prices for both products. As before, the production cost is normalized to zero, and a disclosure cost, δ, is incurred for each product whose quality is disclosed.

Unlike in the duopoly case, the cartel observes both product qualities before making the disclosure decisions, and hence can potentially coordinate over both quality disclosures, based on the realizations of both qualities. Let $q_H = \max\{q_0, q_1\}$ denote the higher of the two qualities and let $q_L = \min\{q_0, q_1\}$ denote the lower of the two qualities. As in the duopoly model, the profit maximizing prices, (p_0, p_1), do not depend on the value of any undisclosed quality. There is no signalling role for prices in our model. Thus, we feel justified in restricting attention to PBE in which consumers' beliefs about undisclosed qualities do not depend on either price. On the other hand, beliefs about, say, q_0, should be allowed to depend on the disclosed value of q_1. However, we want to avoid undesirable equilibria in which a specific high quality is not disclosed when the other (low) quality is not disclosed, supported by the belief that, if the high quality is disclosed, then the undisclosed quality is zero. To resolve this issue, we restrict attention to the following class of symmetric PBE, in which there is a threshold for q_H to be disclosed, and there is a threshold for q_L to be disclosed, which may depend on q_H.

Definition 2: A *cartel equilibrium* is a PBE for which there is a threshold, q^{*C}, and a function, $G(\cdot)$, mapping the unit interval into itself, satisfying

(i) q_H is disclosed if and only if we have $q_H > q^{*C}$,

(ii) q_L is disclosed if and only if q_H is disclosed and we have $q_L > G(q_H)$,

(iii) beliefs are independent of prices,

(iv) if the cartel discloses a quality $q < q^{*C}$, then beliefs are that the undisclosed quality is uniformly distributed over $[0, q]$.

Because of the disclosure cost, no disclosure is always on the equilibrium path of the game. If both qualities are disclosed, beliefs assign probability one to the disclosed qualities. Thus, the only important beliefs off the equilibrium path arise when a quality below the threshold is disclosed, and part (iv) pins down these beliefs in a compelling way. Without loss of generality, let $G(q_H) \leq q_H$ hold. Definition 2 implies the following structure of beliefs in a cartel equilibrium. If nothing is disclosed, consumers believe that both qualities are independently and uniformly distributed over $[0, q^{*C}]$, which follows from Bayes' rule. If only $q_i = q < q^{*C}$ is disclosed, consumers beliefs are determined by Definition 2, part (iv).[12] If only $q_i = q \geq q^{*C}$ is disclosed, consumers believe that the undisclosed quality is the lower quality and is uniformly distributed over $[0, G(q)]$. Since perceived qualities depend on the disclosure choices and are fixed when prices are chosen, the cartel's pricing choice can be represented as a function mapping the perceived qualities into the price of each product.

To characterize the cartel equilibrium, it only remains to identify the pair $(q^{*C}, G(\cdot))$. Given the disclosure strategy characterized by $(q^{*C}, G(\cdot))$, suppose the induced perceived qualities are given by $(\tilde{q}_0, \tilde{q}_1)$. Assuming for the moment that all consumers will be served in equilibrium, a cartel equilibrium must leave the consumer at location x^* indifferent between purchasing product 0, purchasing product 1, and not purchasing. Therefore, the prices must satisfy

(9)
$$\begin{aligned} p_0 &= \psi + \tilde{q}_0 - x^*, \\ p_1 &= \psi + \tilde{q}_1 - (1 - x^*). \end{aligned}$$

The profit maximizing market share is thus given by

$$x^* \in \operatorname{argmax}(\psi + \tilde{q}_0 - x)x + (\psi + \tilde{q}_1 - (1 - x))(1 - x),$$

which results in

(10)
$$x^{*C} = \frac{1}{2} + \frac{1}{4}(\tilde{q}_0 - \tilde{q}_1).$$

From (9) and (10), we derive the prices and cartel profits (gross of any possible disclosure cost), as

[12] Part (iv) says that, when a quality below the threshold is disclosed, the only inference drawn about the undisclosed quality is that it is lower than what was disclosed. The importance of (iv) is to rule out multiple equilibria, based on beliefs that assign a very low mean to the undisclosed quality when $q < q^{*C}$ is disclosed. In effect, this would punish the cartel for disclosing q, which could allow a threshold that is "too high." Any other beliefs would bias the comparison to duopoly.

a function of the perceived qualities, given by:

$$p_0 = \psi - \frac{1}{2} + \frac{3}{4}\tilde{q}_0 + \frac{1}{4}\tilde{q}_1 \tag{11}$$

$$p_1 = \psi - \frac{1}{2} + \frac{1}{4}\tilde{q}_0 + \frac{3}{4}\tilde{q}_1 \tag{12}$$

$$\pi^C(\tilde{q}_0, \tilde{q}_1) = \psi - \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 + \tilde{q}_1) + \frac{1}{8}(\tilde{q}_0 - \tilde{q}_1)^2 \tag{13}$$

It is straightforward to show that $\psi > \frac{3}{2}$ guarantees that the cartel will want to serve all consumers in equilibrium.[13]

The cartel equilibrium is now characterized in Proposition 2 below.

Proposition 2 *There is a unique cartel equilibrium. The disclosure policy is as follows:*

If $\delta < \frac{7}{32}$ holds, we have

$$q^{*M} = 4\sqrt{1 + 2\delta} - 4 \tag{14}$$

and there is a second threshold for q_H, denoted by $\hat{q}$, given by $\hat{q} = 4 - 4\sqrt{1 - 2\delta}$, above which sufficiently high q_L is also disclosed, characterized by the increasing function

$$G(q_H) = \begin{cases} \frac{2q_H - 4}{3} + \frac{2}{3}\sqrt{(q_H)^2 - 4q_H + 4 + 24\delta} & \text{for} \quad q_H \in [\hat{q}, 1] \\ q_H & \text{for} \quad q_H \in [q^{*M}, \hat{q}]. \end{cases} \tag{15}$$

If $\frac{7}{32} \leq \delta < \frac{9}{32}$ holds, we have $q^{*M} = 4\sqrt{1 + 2\delta} - 4$ and the lower quality is never disclosed, so (without loss of generality) let $G(q_H) = q_H$ hold.

If $\frac{9}{32} \leq \delta$ holds, neither quality is disclosed.

Beliefs are as implied by Definition 2, and prices are given by (11) and (12).

Proof: See the appendix.

---

[13] Let x_0 denote the market share of product 0, and let x_1 denote the market share of product 1. If not all consumers are served, so we have $x_0 + x_1 < 1$, then the necessary first order conditions for profit maximization are $x_i = (\psi + \tilde{q}_i)/2$ for $i = 0, 1$. However, no matter what the (nonnegative) perceived qualities are, the first order conditions are inconsistent with $\psi > \frac{3}{2}$ and $x_0 + x_1 < 1$.

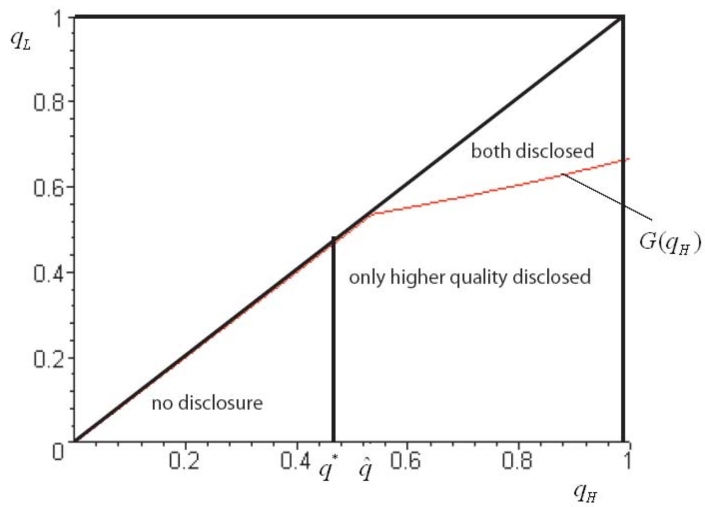

Figure 1: Cartel Equilibrium Disclosure Choices ($\delta = 1/8$)

Proposition 2 shows that the disclosure policy in the cartel equilibrium exhibits some coordination between products, because the decision whether to disclose q_L can be contingent on the value of q_H, as seen from the fact that $G(\cdot)$ varies with q_H. Would a social planner also use such a coordination device? This question will be addressed in the next section, where we look at the effect of market structure on welfare.

V WELFARE ANALYSIS

We now consider the impact of market structure on welfare, which is measured as ex ante expected total surplus (i.e., the sum of producer surplus and consumer surplus). There are two potential sources of inefficiency in our model. One is the misallocation of the products to consumers, and the other is the payment of disclosure costs. Since all consumers are served in the market, we abstract here from 'total output' inefficiencies. We first demonstrate that the expected amount of disclosure is higher under a cartel than under duopoly.

Proposition 3 *In equilibrium, we have $q^{*C} < q^{*D}$, and the expected number of disclosed products under duopoly is strictly lower than under a cartel, for $\delta \in (0, 9/32)$. Outside this range, either both products are disclosed ($\delta = 0$) or neither product is disclosed ($\delta \geq 9/32$) under both market* *structures.*

Proof: See the appendix.

The comparison is illustrated in Figure 2 below, where Δ denotes the expected number of disclosed products.

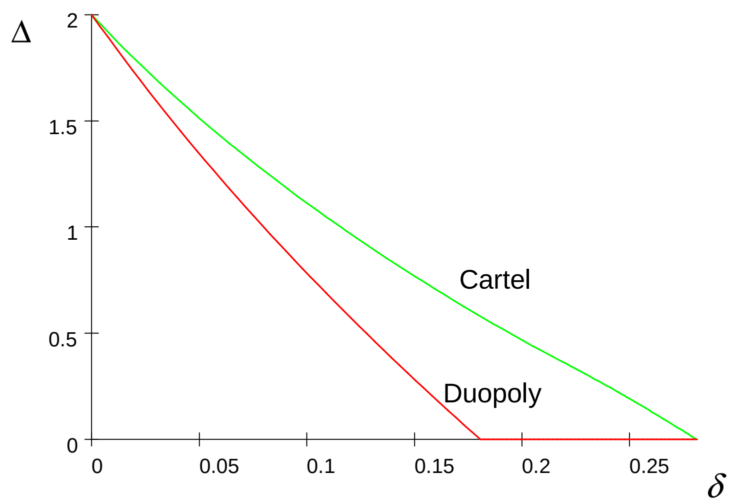

Figure 2: Expected Disclosure

Proposition 3 indicates that, on average, equilibrium disclosure is higher under a cartel than under duopoly. The intuition can be understood as follows. Fix δ, and suppose that the equilibrium thresholds could be the same under duopoly and a cartel, $q^{*D} = q^{*C} \equiv q$. Now suppose that a firm is considering whether to disclose the quality of product 0, where we have $q_0 = q$. (Since we are comparing q^{*D} and q^{*C}, product 0 is taken to be the higher quality product when discussing the cartel case, without loss of generality.) Disclosing quality q causes $\widetilde{q}_0$ to increase by $q/2$. From (2)-(4), we see that, under duopoly, p_0 goes up by $q/6$ and sales of product 0 go up by $q/12$. From (10)-(12), we see that, under a cartel, p_0 goes up by $3q/8$ and sales of product 0 go up by $q/8$. Thus, the price increase and quantity increase are both higher under a cartel, so the increase in revenues for product 0 must be higher.[14] For product 1, sales decrease more under duopoly than under a

[14]This goes against the 'business stealing' intuition, that a duopolist might be more tempted to disclose than a cartel, because the duopolist is not concerned about the other product's profit losses. Here, when a duopolist discloses the

cartel; the price goes down under duopoly and up under a cartel. Overall, the expected benefit is higher under a cartel, contradicting the fact that the threshold equates the expected benefit and the cost, δ. Instead, we must have $q^{*C} < q^{*D}$.[15]

The following proposition compares welfare under the two market structures. When the disclosure cost is relatively small, welfare is higher under a cartel than duopoly. When the disclosure cost is moderate, then welfare is higher under duopoly than a cartel. When the disclosure cost is high enough, so that nothing is disclosed under either market structure, welfare is the same under a cartel and duopoly.

Proposition 4 *There exists $\delta^* \in (0, 13/72)$, such that for $\delta \in [0, \delta^*)$, the expected total surplus is higher under a cartel, and for $\delta \in (\delta^*, 9/32)$, the expected total surplus is higher under duopoly. For $\delta \geq 9/32$, welfare is the same under the two market structures.*

Proof: See the appendix.

Expected total surplus (ETS) under a cartel and duopoly are depicted in Figure 3. The explanation for the U-shaped patterns is best understood by considering expected payments of the disclosure cost. When δ is close to zero, both qualities are likely to be disclosed, but the cost is relatively low. As δ increases, we have less disclosure, but higher payments of the disclosure cost, so welfare goes down. Eventually, we have the surprising result that welfare increases as δ increases: Higher δ in this range leads to less disclosure, which actually improves welfare.

---

other firm lowers its price, thereby counteracting the business stealing. When a cartel discloses it is able to raise the other product's price. Thus, there is more "business shifting" under a cartel than business stealing under duopoly.

[15] This argument explains why $q^{*C} < q^{*D}$ must hold, so the probability of at least one product's quality being disclosed is higher under a cartel. However, the comparison of expected disclosure is more complicated. When the cartel never discloses q_L, then duopolists never disclose at all. When the cartel sometimes discloses q_L (see Figure 1), then it turns out that the probability of both products being disclosed is higher under a cartel than under duopoly. While we conjecture that the result, $q^{*C} < q^{*D}$, is robust to alternative specifications of the model, the comparison of expected disclosure is likely to depend on parameters.

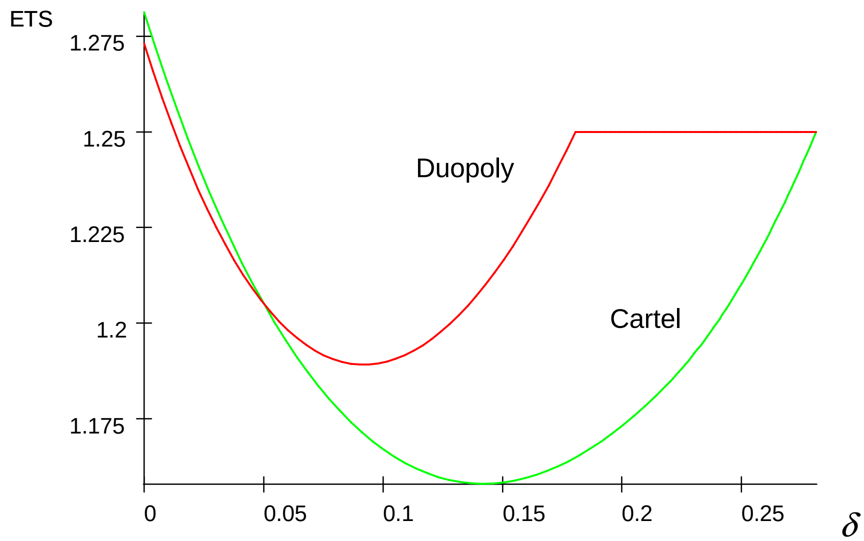

Figure 3: Welfare Comparison ($\psi = 2$)

We now turn to the intuition for the welfare comparison between duopoly and a cartel. Because, as it turns out, equilibrium disclosure is excessive compared to the socially optimal disclosure (see the next section), and this problem is worse for a cartel than duopoly, the loss in welfare due to excessive disclosure costs being incurred favors duopoly over a cartel. The other source of welfare loss is due to the divergence between the actual market share and the optimal market share. Given the perceived qualities of the two products, $\tilde{q}_0$ and $\tilde{q}_1$, the market share for product 0 that maximizes social surplus is easily seen to be[16]

$$x^{**} = \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 - \tilde{q}_1). \tag{16}$$

Comparing x^{**}, x^{*D} from (4), and x^{*M} from (10), we see that the equilibrium market share is closer to the socially optimal market share under a cartel than under duopoly.[17] When δ is near zero, the welfare loss due to excessive disclosure costs being incurred is also near zero, so the misallocation

[16] Because the price is a transfer from consumers to firms, the transaction of the consumer at location x purchasing product 0 adds $\psi + \tilde{q}_0 - x$ to expected total surplus, and the consumer at location x purchasing product 1 adds $\psi + \tilde{q}_1 - (1 - x)$ to expected total surplus. To maximize expected total surplus, x^{**} must equate these two terms.

[17] That is, $|x^{*M} - x^{**}| \leq |x^{*D} - x^{**}|$, where the inequality is strict when $\tilde{q}_0 \neq \tilde{q}_1$.

distortion causes welfare to be higher under a cartel. As δ increases, eventually the welfare loss due to excessive disclosure costs being incurred dominates, and welfare is higher under duopoly. When we have $\delta \geq 9/32$, no disclosure occurs in either market structure. The expected quality and market share of each product is one half under both duopoly and a cartel, so expected total surplus is exactly the same in each market structure.

VI SOCIALLY OPTIMAL DISCLOSURE

In this section we consider the socially optimal policy regarding disclosure. This is of primary interest as a benchmark to compare with the equilibrium, although the analysis may be of some interest to policy makers. Formulating the appropriate planner's problem is tricky. The socially optimal level of disclosure should probably take the market structure as given, rather than allowing the planner to choose prices. If the planner could choose prices, then by letting prices be increasing functions of quality, the planner could indirectly disclose quality without incurring any disclosure costs. Therefore, the planner is assumed to be able to choose disclosure cutoffs but not prices.

The new time line is as follows. First, the social planner picks the disclosure policy (which is q^{**D} under duopoly and $(q^{**C}, G^{**}(\cdot))$ under a cartel). Then the firms disclose according to the policy chosen by the social planner. Then, after quality disclosure, the firms choose prices. Finally, the consumers make their purchase decisions, given the disclosures and prices. The socially optimal disclosure policy is the policy that maximizes ex ante expected total surplus.

Proposition 5 *Under duopoly, the socially optimal disclosure policy is given by* $q^{**D} = 12\sqrt{\delta/5}$ *for* $\delta \in [0, 5/144]$*, and* $q^{**D} = 1$ *for* $\delta > 5/144$*. Under a cartel, the socially optimal disclosure policy is given by* $q^{**C} = G^{**}(q_H) = 8\sqrt{\delta/3}$ *for* $\delta \in [0, 3/64]$*, and no disclosure for* $\delta > 3/64$*. In either market structure, equilibrium disclosure is excessive, as compared to its socially optimal disclosure benchmark.*

Proof: See the appendix.

The following proposition shows that, while equilibrium disclosure is excessive from the standpoint of society, consumer surplus is higher with more disclosure. Thus, our model is consistent with consumer advocates, or agencies whose mission is to protect consumer interests, pushing for mandatory disclosure laws. Specifically, we consider a planner who chooses an arbitrary disclosure threshold, q^D or q^C. Given the threshold, the firms (or cartel) disclose any product whose quality is above the threshold, which determines perceived qualities and prices according to (2), (3), and (9). Therefore, we can determine expected consumer surplus as a function of q^D or q^C.

Proposition 6 *Given the planner's threshold, q^D or q^C, the resulting expected consumer surplus is decreasing in q^D or q^C. In other words, more mandated disclosure increases expected consumer surplus.*

Proof: See the appendix.

Proposition 6 establishes that more disclosure benefits consumers, which is not surprising. The intuition is subtle, though, because when consumers have better information, a cartel might be able to extract more surplus from them. In our cartel model, a consumer's realized surplus is the distance between the consumer's location and x^{*C}. A consumer who always buys product 0 therefore receives an expected surplus equal to the distance between the consumer's location and $\frac{1}{2}$ (the expectation of x^{*C}), independent of the amount of disclosure. However, for a consumer who usually buys product 0 but sometimes switches, more disclosure leads to a wider range of possible x^{*C} values, which implies a greater expected distance between the consumer's location and x^{*C}. Thus, more disclosure implies greater expected surplus. The intuition for duopoly is similar, but more complicated.

The intuition for our over-disclosure result, Proposition 5, has to do with our implicit assumption in our analysis that firms cannot pre-commit to their disclosure policy. To see this, let's first consider the case of a cartel. At the threshold chosen by the planner, q^{**C}, the cartel is better off ex ante with a higher threshold. [This is because consumers prefer a lower threshold, by Proposition 6. Since q^{**C} balances the interests of consumers and the cartel, the cartel (ex ante) must prefer a higher threshold, or less disclosure.] Although the cartel is better off with *less* disclosure than under the planner's benchmark, the cartel's equilibrium strategy is to disclose *more* than under the planner's benchmark! Not only would society be better off by committing to the planner's disclosure choice, but the cartel would receive higher ex ante profits by doing so. For the duopoly case, a similar argument applies: at the threshold chosen by the planner, q^{**D}, both firms are better off ex ante with a higher threshold (recall that we are focusing on symmetric equilibrium in which both equilibrium cutoffs are the same). Whether firms can commit to their disclosure policy or not has substantial welfare implications. In Section 7, we demonstrate that, when firms can commit to their disclosure policy, equilibrium exhibits under-disclosure in both market structures.

Another interesting observation is that the socially optimal disclosure policy under a cartel does not involve coordination between the higher and lower qualities. That is, $G^{**}(q_H)$ does not vary with q_H. One might think that the socially optimal cutoff for disclosing the lower quality would depend on the value of the higher quality, and indeed the cartel equilibrium features such coordination. However, Proposition 5 shows that the planner chooses a product to be disclosed if and only if its quality exceeds $8\sqrt{\delta/3}$. Notice that, although the planner sets a single threshold in either market structure, the threshold for duopoly exceeds the threshold for a cartel. The reason is that the misallocation is larger for duopoly than for a cartel, so the planner is slightly less encouraging of disclosure under duopoly than under a cartel.

VII DISCUSSION

Our results on excessive disclosure rely on the assumption that the disclosure cost is sunk. While this result is consistent with consumers preferring more disclosure, it merits a discussion of how the model might be modified to generate insufficient disclosure. We now show that the welfare results can be reversed if we reinterpret the disclosure cost. We offer two possible interpretations. The first interpretation relates to the behavioral approach taken by Hirshleifer *et al.* (2002). In their model, a fraction of the consumers do not realize that the firm has an opportunity to disclose product quality, unless the disclosure occurs. The resource cost of disclosure is zero, but a firm disclosing low quality lowers the quality perception of naive consumers. As a result, the equilibrium is characterized by a threshold quality for disclosure, which is a function of the fraction of naive consumers. There is a one-to-one correspondence between our disclosure cost δ, and the fraction of naive consumers that would yield the same disclosure threshold. However, with the new interpretation of δ as a reduced-form parameter representing bounded rationality, disclosure represents a loss of revenues to the firms but not a cost to society. With this interpretation, we would have under-disclosure in equilibrium. The motivation for mandating or subsidizing disclosure, then, would be to protect naive consumers from their mistakes.

The second interpretation of the disclosure cost is as a transfer payment to individuals or industries outside the model, part of which does not represent a net cost to society. For example, the disclosure cost could represent rents received by the scientific community or advertising firms. Suppose that the social cost of disclosure is given by $\gamma\delta$, where $\gamma \in [0, 1]$ is a parameter representing the portion of the cost that is sunk. Thus, $\gamma = 1$ corresponds exactly to our main model, and $\gamma = 0$ corresponds to the case in which δ is a transfer from the disclosing firm to other segments of society. With this interpretation, γ affects the welfare calculation but not the equilibrium disclosure choices, pricing choices, or purchasing choices. Therefore, expected total surplus under duopoly, given as (28) for our main model, becomes

$$ETS^D = \psi + \frac{59}{216} - \frac{5}{216}(q^{*D})^3 - 2(1 - q^{*D})\gamma\delta. \tag{17}$$

Maximizing ETS^D over q^{*D}, we have the socially optimal disclosure threshold:

$$q^{**D} = \begin{cases} 12\sqrt{\gamma\delta/5} & \text{for} \quad \gamma\delta \in [0, 5/144] \\ 1 & \text{for} \quad \gamma\delta > 5/144 \end{cases} \tag{18}$$

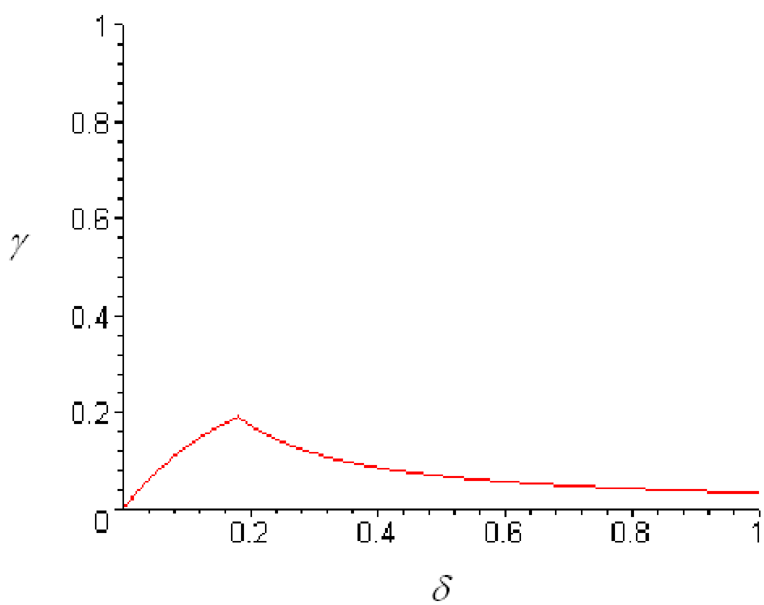

Figure 4: Duopoly Under-disclosure Region

In Figure 4, the area below the curve indicates the set of parameters for which the duopoly equilibrium exhibits under-disclosure. The kink occurs at $\delta = 13/72$, where we have no disclosure in the duopoly equilibrium. Above the curve to the left of the kink, we have excessive disclosure in the duopoly equilibrium, and above the curve to the right of the kink, the duopoly equilibrium has no disclosure, which is optimal. What strikes us about Figure 4 is that the under-disclosure region is quite small. Overturning our result about excessive disclosure seems to require an extreme assumption about the fraction of disclosure costs that represents a net loss to society.[18]

Finally we come back to the issue of commitment. Our framework requires that the firm(s) cannot commit to a disclosure policy. As a result, firms' disclosure policies are optimal given their realized qualities. However, the socially optimal welfare benchmarks are provided from *ex ante* perspective. To illustrate the significance of the inability to commit, we now consider what would happen if firms could commit to their disclosure policies before they observe their types (qualities). Using the subscript c to denote the game with commitment, it can be shown that in the duopoly case, the equilibrium cutoff is given by[19]

$$q_c^{*D} = \begin{cases} \sqrt{72\delta} & \text{for} \quad \delta \in [0, 1/72] \\ 1 & \text{for} \quad \delta > 1/72. \end{cases}$$

In the cartel case, the equilibrium does not involve coordination over both qualities. Thus, the disclosure cutoff is the same for both qualities, which is given by

$$q_c^{*C} = \begin{cases} \sqrt{32\delta} & \text{for} \quad \delta \in [0, 1/36] \\ 1 & \text{for} \quad \delta > 1/36. \end{cases}$$

Comparing with the socially optimal cutoffs stated in Proposition 5, we have

$$q^{**C} \leq q^{**D} \leq q_c^{*C} \leq q_c^{*D}$$

That is, with commitment the equilibrium disclosure is insufficient rather than excessive in both market structures. Further analysis shows that with such commitment, the welfare comparison

[18] The cartel case is similar, but complicated by the fact that disclosure in the cartel equilibrium cannot be characterized by a single threshold.

[19] The proof of this case and that of the cartel case below, are available upon request.

between a cartel and duopoly is no longer bifurcated in δ. In fact, the expected total surplus is always higher in the cartel case than in the duopoly case.[20]

Thus, commitment leads to dramatic changes in the welfare implications of our model. In the cartel case without commitment, the cartel does not coordinate between its possible types when making the disclosure decision: when the realized type is between the cartel equilibrium threshold and the planner's threshold, $q^{*C} < q_H < q^{**C}$, the cartel's disclosure improves its realized profits, but harms its lower quality types by more than the amount it benefits consumers, leading to lower ex ante expected surplus as well as its own profit. With commitment, the cartel can coordinate between its possible types ex ante in deciding on the disclosure cutoffs. As a result, the ex ante expected profit (and expected surplus) both increase. In the duopoly case, a similar intuition applies, though the argument is a bit more subtle.

VIII CONCLUSION

Our framework has several advantages over the previous literature on disclosure. Much of that literature fails to treat quality as private information, or only considers the case of monopoly. The remaining papers complicate the comparison of duopoly and monopoly, by assuming that each firm produces one product. Thus, increasing the number of firms changes the market structure, but it also changes the technology by affecting the number of draws from the quality distribution. Putting the duopoly and cartel models together, while holding the joint distribution of qualities fixed, allows for a sensible welfare comparison and generates fresh insights. Due to our well 'controlled' analytical benchmarks, we are able to perform a clean analysis of the effect of competition on equilibrium disclosure and welfare.

Generalizing the model beyond duopoly is not straightforward, because that would introduce asymmetries in the distribution of consumer preferences over the unit interval. We believe that considering more general distributions of product quality would be tractable and worthwhile. Our independent, uniform assumption allows us to compute solutions and identify regularities that would

[20] Although equilibrium disclosure is insufficient in both market structures, a cartel discloses more than duopoly. Thus the welfare loss due to insufficient disclosure is alleviated in the cartel case. This, combined with the fact that a cartel continues to induce less distortion in product allocation, implies that the expected welfare is higher in the cartel case regardless of the magnitude of δ.

have been difficult to discover otherwise. However, our goal in future work is to explore the implications of correlated quality across products.

APPENDIX

Proof of Proposition 2:

We first show that $(q^{*C}, G(\cdot))$, as specified in the statement of the proposition, characterizes a cartel equilibrium. Given the cartel's strategy and given beliefs, it is clear that consumer behavior, as determined by x^* in (10), is sequentially rational.[21] Given the disclosure choices and consumer behavior, the prices given in (11) and (12) are constructed to uniquely satisfy sequential rationality. As for the disclosure choices, the gross profit expression in (13) is increasing in perceived qualities, so it is never rational for the cartel to disclose q_L but not q_H.[22] Consider the following three remaining possibilities.

1. Neither product is disclosed.

   By (13), the cartel profit, as a function of the two qualities, is given by

   (19) $$\pi_1(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}q^{*C}.$$

2. Only q_H is disclosed.

   By (13), the cartel profit is given by

   (20) $$\pi_2(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}(q_H + G(q_H)/2) + \frac{1}{8}(q_H - G(q_H)/2)^2 - \delta.$$

3. Both products are disclosed.

[21] Equation (8) is based upon the cartel choosing prices that serve the entire market and extract all the surplus from the marginal consumer. If the cartel chooses prices that leave all consumers with positive surplus, or prices that leave some consumers strictly preferring not to purchase, the unit interval should be partioned in the obvious way to ensure consumer rationality off the equilibrium path.

[22] Without Definition 2, part (iv), the cartel might benefit by disclosing the lower quality, if that could increase the perceived quality of the undisclosed product. However, part (iv) and the fact that G is increasing implies that the perceived quality of the undisclosed product is increasing in the quality of the disclosed product.

By (13), the cartel profit is given by

$$\pi_3(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}(q_H + q_L) + \frac{1}{8}(q_H - q_L)^2 - 2\delta. \tag{21}$$

The threshold, $q^{*C} = 4\sqrt{1+2\delta} - 4$, guarantees that, when $q_H = q^{*C}$ holds, the cartel is indifferent between disclosing neither product and disclosing q_H. To see this, substitute $q_H = G(q_H) = q^{*C}$ into (20) and (19), equate π_2 with π_1, and solve for q^{*C}. If $\delta < \frac{9}{32}$ holds, then we have an interior threshold with $q^{*C} < 1$. If $\delta \geq \frac{9}{32}$ holds, then the cartel always prefers not to disclose, and $q^{*C} = 1$ ensures that nothing is disclosed. Because π_2 is strictly increasing in q_H, the cartel strictly prefers not to disclose anything when q_H is below the threshold, and strictly prefers to disclose q_H (and possibly q_L as well) when q_H is above the threshold.

It remains to show that the decision whether to disclose q_L, based on $G(\cdot)$, is also consistent with sequential rationality. Suppose $\delta < \frac{7}{32}$ holds. If $q_H > q^{*C}$ is disclosed, and we have $\hat{q} \leq q_H \leq 1$ and

$$q_L = \frac{2q_H - 4}{3} + \frac{2}{3}\sqrt{(q_H)^2 - 4q_H + 4 + 24\delta},$$

then the cartel is indifferent between disclosing q_L and not disclosing it. To see this, substitute $q_L = G(q_H)$ in (21), equate π_3 with π_2, and solve for $G(q_H)$. We specified $\hat{q}$ to solve

$$q_H = \frac{2q_H - 4}{3} + \frac{2}{3}\sqrt{(q_H)^2 - 4q_H + 4 + 24\delta},$$

so $\hat{q} \leq q_H \leq 1$ implies there is a range of q_L that will be disclosed. The condition, $\delta < \frac{7}{32}$, guarantees that $\hat{q} < 1$ holds, so the interval, $\hat{q} \leq q_H \leq 1$, is well defined and nondegenerate. It is easily verified that π_3 is strictly increasing in q_L, so the cartel strictly prefers not to disclose q_L when $q_L < G(q_H)$ holds, and strictly prefers to disclose q_L when $q_L > G(q_H)$ holds.

If $q_H > q^{*C}$ is disclosed, and we have $q^{*C} \leq q_H \leq \hat{q}$, then the cartel strictly prefers not to disclose q_L for all $q_L \leq q_H$, so not disclosing q_L is sequentially rational.

Now suppose $\delta \geq \frac{7}{32}$ holds. Since $\hat{q} = 4 - 4\sqrt{1-2\delta} \geq 1$ must hold, then for all $q_H \in [q^{*C}, 1]$, the cartel strictly prefers not to disclose q_L for all $q_L \leq q_H$. Again, not disclosing q_L is sequentially rational. This establishes that the candidate is a cartel equilibrium.

To show that there is a unique PBE consistent with Definition 2, suppose that $(q^{*C}, G(\cdot))$ is different from that specified in Proposition 2. We first show that $G(q^{*C}) < q^{*C}$ is impossible, so as q_H crosses the threshold, disclosing both qualities cannot be optimal. If $G(q^{*C}) < q^{*C}$ were to hold, then if the quality realizations are given by $q_H = q^{*C}$ and $q_L = G(q^{*C})$, the cartel must be indifferent between disclosing nothing and disclosing both qualities. From (19) and (21), we have

$$2\delta = \frac{q_L}{2} + \frac{1}{8}(q_H - q_L)^2. \tag{22}$$

For this to be consistent with equilibrium, the cartel cannot be better off disclosing only q_H. From (19) and (20), we have

$$\delta \geq \frac{q_L}{4} + \frac{1}{8}(q_H - \frac{q_L}{2})^2. \tag{23}$$

By using (23) to substitute for δ in (22), we derive

$$q_H(1 - \sqrt{2}) \geq q_L(1 - \frac{\sqrt{2}}{2}),$$

which is clearly impossible. Since disclosing both qualities cannot be optimal as q_H crosses the threshold, q^{*C} must be such that the cartel is indifferent between disclosing neither product and disclosing q_H when $q_H = q^{*C}$ holds and beliefs are that the undisclosed quality is uniformly distributed over $[0, q^{*C}]$. From the argument above equating π_2 with π_1, $q^{*C} = 4\sqrt{1+2\delta} - 4$ is the unique threshold consistent with a cartel equilibrium. Similarly, $G(q_H)$ is uniquely determined by either indifference between disclosing q_L and not disclosing it (when $\hat{q} \leq q_H \leq 1$ holds), or strict preference not to disclose q_L (when $q^{*C} \leq q_H \leq \hat{q}$ holds).[23] Thus, sequentially rational disclosure choices are uniquely determined. As argued above, given the disclosure choices, beliefs are uniquely determined, and sequentially rational pricing decisions are uniquely determined, given those beliefs. Finally, consumer behavior is uniquely determined as well. ■

Notation for Welfare Computations.

Under a cartel, the expression for welfare is extremely messy. To save on notation, we define the following:

$$\begin{aligned} A &= \sqrt{5 - 2\delta - 4\sqrt{1-2\delta}}, B = \sqrt{1-2\delta}, \\ C &= \sqrt{1+2\delta}, \text{and } D = \sqrt{1+24\delta}. \end{aligned} \tag{24}$$

[23] Without our convention, $G(q_H) \leq q_H$, there would be many $G(\cdot)$ functions consistent with a cartel equilibrium, since the behavior of $G(\cdot)$ above the 45 degree line is arbitrary. However, all such functions lead to the same disclosure choices.

Proof of Proposition 3:

It is straightforward to verify that $q^{*C} < q^{*D}$ holds for $\delta \in (0, 9/32)$. We compute the expected number of disclosed products in the two cases below.

1. Duopoly Case

   For $\delta \in [0, 13/72]$, the probabilities of zero, one, and two products being disclosed are, respectively, $(q^{*D})^2$, $2q^{*D}(1 - q^{*D})$, and $(1 - q^{*D})^2$, where $q^{*D} = -\frac{5}{3} + \frac{1}{3}\sqrt{25 + 216\delta}$.

   The expected number of disclosed products under duopoly, Δ^D, is therefore given by

   $$\Delta^D = \frac{16}{3} - \frac{2}{3}\sqrt{25 + 216\delta}. \tag{25}$$

   For $\delta \geq 13/72$, we have no disclosure with probability one, and $\Delta^D = 0$.

2. Cartel Case

   For $\delta \in [0, 7/32]$, the probabilities of zero, one, and two products being disclosed are, respectively,

   $$\begin{aligned}
   (q^{*C})^2 &= (4\sqrt{1 + 2\delta} - 4)^2, \\
   2\int_{q^{*C}}^{1}\int_{0}^{G(q_H)} dq_L\, dq_H &= \frac{1}{3}[-38 - 128\delta - 8A + 16AB + 96C - 2D] + 16\delta \ln\left[\frac{D - 1}{2(1 + A - 2B)}\right], \\
   2\int_{q^{*C}}^{1}\int_{G(q_H)}^{q_H} dq_L\, dq_H &= \frac{1}{3}[-55 + 32\delta + 8A + 64B - 16AB] - 16\delta\left[\frac{D - 1}{2(1 + A - 2B)}\right].
   \end{aligned}$$

   The expected number of disclosed products, Δ^C, can be computed, as

   $$\Delta^C = \frac{1}{3}[-148 - 64\delta + 8A + 64B - 16AB + 96C + 2D] - 16\delta\left[\ln \frac{D - 1}{2(1 + A - 2B)}\right].$$

   For $\delta \in (7/32, 9/32]$, the cartel never discloses the lower quality. The probabilities of zero and one product being disclosed are, respectively, $(4\sqrt{1 + 2\delta} - 4)^2$, and $1 - (4\sqrt{1 + 2\delta} - 4)^2$. Therefore, we have $\Delta^C = 1 - (-4 + 4\sqrt{1 + 2\delta})^2$. For $\delta > 9/32$, there is no disclosure, and we have $\Delta^C = 0$.

The comparison shows that $\Delta^D > \Delta^C$ holds for all $\delta \in (0, 9/32)$, which is illustrated in Figure 2. ■

Proof of Proposition 4:

We first derive the expected total surplus functions under both market structures.

1. Duopoly Case

Following the equilibrium characterization in Proposition 1, we consider four regions in the space of quality realizations:

(a) $q_0, q_1 < q^{*D}$, neither quality is disclosed.

By (4), in this case the marginal consumer type, who is indifferent between product 0 and product 1, is given by $x^* = \frac{1}{2}$. For a type-x consumer, the surplus from purchasing product 0 is given by $\psi + q_0 - x$, and similarly, the surplus from purchasing product 1 is given by $\psi + q_1 - (1 - x)$. (The price paid is a transfer of surplus from consumers to firms.) Integrating over all consumers, we have the total surplus:

$$ts_1(q_0, q_1) = \int_0^{1/2} (\psi + q_0 - x)\,dx + \int_0^{1/2} (\psi + q_1 - x)\,dx$$

Integrating $ts_1(q_0, q_1)$ over the set of qualities with $q_0, q_1 < q^{*D}$, we have the contribution of this region to expected surplus:

$$ETS_1 = \int_0^{q^{*D}} \int_0^{q^{*D}} ts_1(q_0, q_1)\,dq_1\,dq_0.$$

(b) $q_0 > q^{*D} > q_1$. Only q_0 is disclosed.

By (4), the marginal consumer type is given by

$$x^* = \frac{1}{2} + \frac{1}{6}(q_0 - q^{*D}/2). \qquad (26)$$

Given (q_0, q_1), the total surplus generated is given by

$$ts_2(q_0, q_1) = \int_0^{x^*} (\psi + q_0 - x)\,dx + \int_0^{x^*} (\psi + q_1 - x)\,dx - \delta,$$

where x^* is given by (26). Integrating again to get the contribution of this region to expected total surplus, we have

$$ETS_2 = \int_{q^{*D}}^{1} \int_0^{q^{*D}} ts_2(q_0, q_1)\,dq_1\,dq_0.$$

(c) $q_1 > q^{*D} > q_0$, so q_1 is disclosed but q_0 is not.

By symmetry, we have $ETS_3 = ETS_2$.

(d) $q_0, q_1 > q^{*D}$. Both q_0 and q_1 are disclosed.

By (4), the marginal consumer type is given by

$$x^* = \frac{1}{2} + \frac{1}{6}(q_0 - q_1). \tag{27}$$

Given (q_0, q_1), the total surplus generated is given by

$$ts_4(q_0, q_1) = \int_0^{x^*} (\psi + q_0 - x)\, dx + \int_0^{x^*} (\psi + q_1 - x)\, dx - 2\delta,$$

where x^* is given by (27). Integrating again to get the expected total surplus in this region, we have

$$ETS_4 = \int_{q^{*D}}^{1} \int_{q^{*D}}^{1} ts_2(q_0, q_1)\, dq_1\, dq_0.$$

Putting all these expressions together, the expected total surplus in equilibrium is given by:

$$ETS^D = ETS_1 + 2ETS_2 + ETS_4 = \psi + \frac{59}{216} - \frac{5}{216}(q^{*D})^3 - 2(1 - q^{*D})\delta. \tag{28}$$

Substituting the value of q^{*D}, we have for $\delta < 13/72$,

$$ETS^D = \psi + \frac{4093}{5832} - \frac{23}{9}\delta - (\frac{125}{1458} - \frac{13}{27}\delta)\sqrt{25 + 216\delta}. \tag{29}$$

For $\delta \geq 13/72$, we have $q^{*D} = 1$, and hence $ETS^D = \psi + 1/4$.

2. Cartel Case

Following the equilibrium characterization in Proposition 2, we consider three regions in the space of quality realizations:

(a) $q_L \leq q_H < q^{*C}$, neither quality is disclosed.

By (10), the market share is given by $x^* = 1/2$, and the total surplus given (q_L, q_H) is

$$ts_1(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x)\, dx + \int_0^{1-x^*} (\psi + q_L - x)\, dx.$$

Integrating over this region, we have the contribution to expected total surplus, given by

$$ETS_1 = 2\int_0^{q^{*C}} \int_0^{q_H} ts_1(q_H, q_L)\, dq_L\, dq_H. \tag{30}$$

(b) $q_H > q^{*C}$ but $q_L < G(q_H)$ hold, so only q_H is disclosed.

By (10), the market share is given by $x^* = 1/2 + (q_H - G(q_H)/2)/4$, and the total surplus given (q_L, q_H) is

$$ts_2(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x)\, dx + \int_0^{1-x^*} (\psi + q_L - x)\, dx - \delta.$$

Integrating over this region, we have the contribution to expected total surplus, given by

$$ETS_2 = 2 \int_{q^{*C}}^{1} \int_0^{G(q_H)} ts_2(q_H, q_L)\, dq_L\, dq_H. \tag{31}$$

(c) $q_H > q^{*C}$ and $q_L > G(q_H)$ hold, so both q_H and q_L are disclosed.

By (10), the market share is given by $x^* = 1/2 + (q_H - q_L)/4$, and the total surplus given (q_L, q_H) is

$$ts_3(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x)\, dx + \int_0^{1-x^*} (\psi + q_L - x)\, dx - 2\delta.$$

Integrating over this region, we have the contribution to expected total surplus, given by

$$ETS_3 = 2 \int_{q^{*C}}^{1} \int_{G(q_H)}^{q_H} ts_3(q_H, q_L)\, dq_L\, dq_H. \tag{32}$$

The expected total surplus, ETS^C, is given by $ETS^C = ETS_1 + ETS_2 + ETS_3$.

Substituting the values of q^{*C} and the appropriate expression for $G(q_H)$ into (30), (31) and (32), we can obtain the expected total surplus under a cartel.

For $\delta \in [0, 7/32]$, we have

$$\begin{aligned} ETS^C \;=\; & \psi - \frac{49813}{864} - \frac{673}{27}\delta - (\frac{320}{27} + 16\ln 2)\delta^2 + \frac{-56 + 160\delta}{27} AB + (\frac{52}{27} - \frac{16}{3}\delta)A \\ & + (\frac{272 - 896\delta}{27})B + 16(3 - \delta)C + (\frac{1}{108} - \frac{4}{9}\delta)D + 16\delta^2 \ln\left[\frac{D - 1}{1 + A - 2B}\right]. \end{aligned}$$

For $\delta \in (7/32, 9/32]$, we have

$$ETS^C = \psi - \frac{6109}{128} - 65\delta + 8\delta^2 + 16(3 + \delta)\sqrt{1 + 2\delta}.$$

For $\delta > 9/32$, we have

$$ETS^C = \psi + \frac{1}{4}.$$

Straightforward (but tedious) algebra shows that $ETS^D - ETS^C$ is strictly increasing in δ over $\delta \in [0, 13/72]$. Since $ETS^D - ETS^C$ is negative at $\delta = 0$ and positive at $\delta = 13/72$, by the mean value theorem, there exists a unique $\delta^* \in (0, 13/72)$ such that $ETS^D = ETS^C$ holds. It is also easily verified that we have $ETS^D - ETS^C > 0$ for $\delta \in [13/72, 9/32)$. For $\delta \geq 13/72$, we have $ETS^D = ETS^C = \psi + 1/4$. ■

Proof of Proposition 5:

1. Duopoly Case

   The social planner announces and enforces the disclosure policy characterized by q^*. By (28) the expected total surplus is given by

$$ETS = \psi + \frac{59}{216} - 2\delta + 2\delta q^* - \frac{5}{216} q^{*3}.$$

   Therefore the socially optimal cutoff is given by

$$\begin{aligned} q^{**D} &= 12\sqrt{\delta/5} \text{ for } \delta \in [0, \frac{5}{144}], \\ q^{**D} &= 1 \text{ for } \delta \geq \frac{5}{144}. \end{aligned}$$

   It is easy to see that we have $q^{**D} > q^{*D}$ for all $\delta \in (0, 13/72)$ and $q^{**D} = q^{*D}$ for $\delta = 0$ and $\delta > 13/72$. Therefore, in the duopoly equilibrium, there is too much disclosure compared to the socially optimal disclosure.

2. Cartel Case

   The social planner announces and enforces the disclosure policy characterized by $(q^*, G(\cdot))$. Let $(\widetilde{q}_H, \widetilde{q}_L)$ be the perceived qualities about (q_H, q_L) induced by the cartel's disclosure announcement. Also let x^* denote the consumer type that is indifferent between purchasing the higher quality product and the lower quality product. Then we have

$$x^* = \frac{1}{2} + \frac{1}{4}(\widetilde{q}_H - \widetilde{q}_L)$$

   Following the same steps as in the proof of Proposition 4, we can write the expected total surplus, given the disclosure rule $(q^*, G(\cdot))$, as follows.

(33)

$$ETS = 2\int_0^{q^*}\int_0^{q_H}(ts_1(q_H, q_L)\,dq_L, dq_H + 2\int_{q^*}^1 \underbrace{[\int_0^{G(q_H)} ts_2(q_H, q_L)\,dq_L + \int_{G(q_H)}^{q_H} ts_3(q_H, q_L)\,dq_L]}_{=\Omega(G)}\,dq_H$$

where ts_1, ts_2 and ts_3 are all given in the proof of Proposition 4 for the cartel case.

The optimal $G(\cdot)$ maximizes $\Omega(G)$ for any given q_H. Differentiating yields

$$\begin{aligned} G^{**}(q_H) &= q_H \text{ for } q_H < 8\sqrt{\delta/3} \\ G^{**}(q_H) &= 8\sqrt{\delta/3} \text{ for } q_H \geq 8\sqrt{\delta/3} \end{aligned} \tag{34}$$

Substituting $G(\cdot) = G^{**}(\cdot)$ into (33), and then differentiating with respect to q^*, we have

$$\frac{dETS}{dq^*} = -\frac{3}{32}(q^*)^3 + 2\delta q^*.$$

The optimal q^* is thus given by

$$\begin{aligned} q^{**C} &= 8\sqrt{\delta/3} \text{ for } \delta \in [0, 3/64] \\ q^{**C} &= 1 \text{ for } \delta > 3/64. \end{aligned} \tag{35}$$

Combining (34) and (35), the social planner's disclosure policy is given below:

$$q^{**C} = \begin{cases} 8\sqrt{\delta/3} & \text{if } \delta \in [0, 3/64] \\ 1 & \text{if } \delta > 3/64 \end{cases}$$

and $G^{**}(q_H) = 8\sqrt{\delta/3}$ for $q_H \in (q^{**C}, 1]$.

Since $G^{**}(q_H)$ does not vary with q_H, in effect the socially planner sets a uniform threshold for both quality disclosures. In other words, the potential coordination instrument provided by $G(\cdot)$ does not help to improve social welfare.

We have $q^{**C} > q^{*C}$ for all $\delta \in (0, 9/32)$ and $q^{**C} = q^{*C}$ for $\delta = 0$ and $\delta > 9/32$. It can also be verified that $G^{**}(q_H) \geq G(q_H)$ holds for any q_H. Therefore, in the cartel equilibrium, there is too much disclosure as compared to the socially optimal disclosure. ■

Proof of Proposition 6:

Under duopoly, given q^D, a product with quality $q_i < q^D$ has perceived quality $\widetilde{q}_i = q^D/2$. Equations (2) and (3) determine prices for each pair of quality realizations, and (4) determines which consumers buy which product. This allows us to write consumer surplus as a function of the realized qualities.[24] Integrating over the quality distribution, we have expected consumer surplus,

$$ECS^D = \psi - \frac{1}{216}(q^D)^3 - \frac{161}{216},$$

from which the result follows.

Under a cartel, given q^C, a product with quality $q_i < q^C$ has perceived quality $\widetilde{q}_i = q^C/2$. Equation (9) determines prices for each pair of quality realizations, and (10) determines which consumers buy which product. This allows us to write consumer surplus as a function of the realized qualities. Integrating over the quality distribution, we have expected consumer surplus,

$$ECS^C = -\frac{1}{96}(q^C)^3 + \frac{25}{96},$$

from which the result follows. ■

REFERENCES

Board, O., 2003, 'Competition and the impact of mandatory disclosure laws', Working Paper.

Cheong, I. and Kim, J., 2004, 'Costly information disclosure in oligopoly', *Journal of Industrial Economics*, 1, 121-132.

Daughety, A. F. and Reinganum, J. F., 2005, 'Secrecy and Safety', *American Economic Review*, 95, pp. 1074-1091.

Daughety, A. F. and Reinganum, J. F., 2007, 'Competition and Confidentiality: Signaling Quality in a Duopoly When There is Universal Private Information', *Games and Economic Behavior,* 58, pp. 94-120.

Dye, R.A. and Sridhar, S.S., 1995, 'Industry-wide Disclosure Dynamics', *Journal of Accounting Research,* 33, pp. 157-174.

[24] The expressions in each of the four regions of quality space are omitted to save on space.

Fishman, M.J. and Hagerty, K.M., 2003, 'Mandatory versus voluntary disclosure in markets with informed and uninformed customers', *Journal of Law, Economics, and Organization,* 19, pp. 45-63.

Grossman, S. J. and Hart, O.D., 1980, 'Disclosure laws and takeover bids', *Journal of Finance,* 35, pp. 323-334.

Hirshleifer, D., Lim, S.S., and Teoh, S.H., 2002, 'Disclosure to a credulous audience: the role of limited attention', Ohio State University Working Paper.

Hotz, V.J. and Xiao, M., 2005, 'Strategic Information Disclosure: The Case of Multi-Attribute Products with Heterogeneous Consumers', Working Paper.

Jovanovic, B., 1982, 'Truthful disclosure of information', *Bell Journal of Economics,* 13, pp. 36-44.

Matthews, S. and Postlewaite, A., 1985, 'Quality Testing and Disclosure', *Rand Journal of Economics,* 16, pp. 328-334.

Milgrom, P.R. and Roberts, J., 1986, 'Price and advertising as signals of product quality', *Journal of Political Economy,* 66, pp. 215-232.

Stivers, A., 2004, 'Unraveling of Information: Competition and Uncertainty', *The B.E. Journals: Topics in Theoretical Economics*, 4(1).

Okuno-Fujiwara, M., Postlewaite, A. and Suzumura, K., 1990, 'Strategic information revelation', *Review of Economic Studies*, 57, pp. 25-47.

Viscusi, W.K., 1978, 'A note on "Lemons" markets with quality certification', *Bell Journal of Economics*, 9, pp.277-79.