Search with Learning: Understanding Asymmetric Price Adjustments

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Abstract

In many retail markets, prices rise faster under positive cost shocks than they fall under negative shocks. We develop a model of search with learning to explain this phenomenon of asymmetric price adjustments. In the static game we show that there is a unique equilibrium in which the search intensity decreases as the consumers' belief about the high-cost state increases. By extending our analysis to the dynamic setting where the cost evolves according to a Markov process with positive persistence, we demonstrate that asymmetric price adjustments arise naturally. When a positive cost shock occurs, all the searchers immediately learn the true state, the search intensity, and hence the prices fully adjust in the next period. When a negative cost shock occurs, it takes longer for non-searchers to learn the true state, and the search intensity increases gradually, leading to a slow falling of prices.

Keywords: Asymmetric price adjustment, Rockets and feathers, Search, Learning

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1 Introduction

Firms are quick to raise prices in response to their cost increases, but not so keen to reduce prices when their costs fall. This widespread phenomenon is known as asymmetric price adjustment, or the *rockets* and *feathers*. This pattern of asymmetric price adjustment has been reported in a broad range of product markets. In fact, a growing empirical literature documents asymmetric price adjustment in various product markets, including gasoline (Bacon, 1991; Karrenbrock, 1991; Duffy-Deno, 1996; Borenstein, Cameron and Gilbert, 1997; Eckert, 2002; and Deltas, 2004), fruit and vegetables (Pick et al., 1991; and Ward, 1982), beef and pork (Boyd and Brorsen, 1988; Goodwin and Holt, 1999; and Goodwin and Harper, 2000), and banking (Hannan and Berger, 1991; Neumark and Sharpe, 1992; and O'Brien, 2000).[1] According to Peltzman (2000), asymmetric price adjustment is found in more than two of every three markets examined in a large sample with 77 consumer goods and 165 producer goods.

Despite these extensive empirical studies confirming the general pattern of asymmetric price adjustment, there is little theoretical work examining this phenomenon. In fact, asymmetric price adjustment first appeared to be inconsistent with conventional microeconomic theory, which usually suggests that an increase or decrease of input prices should affect the marginal costs, and hence move the prices up or down in a symmetric, rather than asymmetric way. As Peltzman (2000) put it, the "stylized fact" of asymmetric price adjustment "poses a challenge to theory." This paper attempts to help bridge such a gap in the literature.

More specifically, we develop a model of search with learning in a dynamic framework. We start with a description of the static game. There are a continuum of consumers and a continuum of firms with capacity constraints. Firms have a common unit production cost (either high or low). While known to the firms, the cost is unknown to the consumers. There are three types of consumers: the low search cost consumers who always search, the high search cost consumers who never search, and critical consumers whose search cost is intermediate. The decision for a critical consumer to search or not depends on whether the expected benefit of searching outweighs her search cost, so the percentage of consumers who search (the search intensity) will be endogenously determined. We adopt the protocol of non-sequential search, that is, consumers who search observe the prices charged

[1] It is found that deposit rates respond more quickly to an increase of money market rates than to a decrease of market rates.

by all firms, so searchers always shop at firms with the lowest price available (unless they are rationed due to firms' capacity constraint, in which case they will shop at the firms with the next lowest price, and so forth). On the other hand, non-searchers shop randomly and only observe one price.

In the static game, we show that there is a unique equilibrium. Critical consumers hold heterogenous beliefs regarding the cost state, and the equilibrium search intensity only depends on critical consumers' distribution of initial beliefs. As more consumers' initial beliefs about the high-cost state lie below some cutoff level, the equilibrium search intensity increases. This is because prices are more dispersed when the cost is low due to competition among firms, leading to a higher expected gain from search. The equilibrium price distribution depends on the search intensity and the actual cost state. Specifically, the equilibrium prices are increasing in the actual cost, and are decreasing in search intensity, since each firm's demand becomes more elastic as more consumers search. Thus the full adjustment of equilibrium prices requires the adjustment of search intensity, which solely depends on the critical consumers' belief updating process.

We then extend our static game analysis to a dynamic setting where the cost evolves according to a Markov process with positive persistence. Since consumers never observe the cost realizations, each consumer updates her belief based on the history of prices she observed. Thus consumers have heterogenous beliefs due to the different histories of prices they observe. In equilibrium searchers and non-searchers have different belief updating processes. Searchers always correctly learn the true state, since they always observe the lowest price that fully reveals the true state. But non-searchers do not always learn the true state.

Asymmetric price adjustment thus arises naturally. In the event of positive cost shocks, all the searchers among the critical consumers immediately learn the true state and stop searching. In the following period no critical consumers search and the search intensity is the lowest possible. Thus the search intensity and hence the prices fully adjust within two periods. In the event of negative cost shocks, it takes longer for critical consumers who do not search originally, to learn the true state and start searching, thus the search intensity increases gradually, leading to a slow falling of prices. To sum up, asymmetric price adjustment is caused by learning asymmetry between searchers and non-searchers, which is closely related to the evolution of search intensity.

More formally, we show that given the evolution of the underlying cost states, there is a unique equilibrium for the dynamic game, with the evolutions of the distribution of beliefs, the search intensity and the prices uniquely determined. When a positive cost shock occurs, critical consumers stop searching in the next period, and the prices are fully adjusted upward within two periods. On the other hand, when a negative cost shock occurs, critical consumers begin to search gradually, and the prices adjust downward slowly over a longer periods of time. As long as the cost shocks are persistent, this pattern of asymmetric price adjustments emerge in statistical sense. Moreover, as the cost shocks become more persistent, the pattern of asymmetric price adjustments becomes more prominent, because the downward price adjustment on average spreads over a longer periods of time. This is a testable implication.

Several recent papers (Lewis, 2005; Tappata, 2006; and Cabral and Fishman, 2006) also attempt to explain asymmetric price adjustment based on search models.[2] Lewis (2005) develops a *reference price* consumer search model in which consumers' expected prices are based on prices observed during previous purchases. However, the expected distribution of prices are exogenously given, rather tan endogenously determined. Thus consumers are not rational in his model, which is different from our model.

Cabral and Fishman (2006) develop a search model in which the cost changes are positively but not perfectly correlated across firms.[3] They show that consumers have a greater incentive to search in the case of large price increases or small price decreases, but little incentive to search when prices increase a little or decrease by a lot. This implies that firms are reluctant to change prices when costs decrease by a little bit or increase a lot, but quick to change prices as costs increase by a little bit or decrease by a lot. These implications are quite different from ours.

The paper that is most closely related to ours is Tappata (2006). However, our paper is different from Tappata in two major aspects. First, while Tappata assumes that the firms' past costs are known to the consumers, we do not impose this assumption in our analysis. Since consumers know the past costs, there is no learning in Tappata (2006). However, learning about the underlying cost through search plays a major role in our analysis. Second, since past costs are observable to consumers in Tappata, it takes exactly two periods for prices to fully adjust to both the positive and

[2] Borenstein et.al (1997) proposed an explanation for asymmetric price adjustments based on Rotemberg and Saloner's (1986) model of tacit collusion with stochastic shocks.

[3] Specifically, if costs change, then both firms' costs either increase or decrease, though the magnitude of the cost changes might be different between two firms.

negative cost shocks. In contrast, we endogenize the time periods that are needed for prices to fully adjust to cost shocks: while it takes two periods for prices to fully adjust to positive cost shocks, it takes much longer periods for prices to fully adjust in response to negative cost shocks. Thus our model can explain why prices adjust to positive shocks *faster* than to negative shocks.

Our paper also contributes to the literature on consumer search, in that we developed a dynamic search model with consumers, in a heterogenous fashion, learning about the underlying states based on the personal histories of prices they observed.[4] Several previous papers have studied equilibrium search with learning (Benabou and Gertner, 1993; Dana, 1994; Fishman, 1996). Benabou and Gertner (1993) study how the correlations among firms' cost shocks affect consumers' incentive to search and the equilibrium prices. In a static model, Dana (1994) shows that if consumers are uncertain about firms' costs, then the response of prices to cost shocks will be limited. In a dynamic framework, Fishman (1996) shows that cost shocks have different short run and long run effects on prices.[5] None of these papers study asymmetric price adjustments.

The paper is organized as follows. Section 2 presents the static game and characterizes the unique static game equilibrium. We extend the static game analysis to the multiple period setting in Section 3, 1and demonstrate that asymmetric pricing arises naturally in equilibrium. Section 4 offers some discussion and Section 5 concludes.

2 Static Game

2.1 The Model

We consider a market with a continuum of firms producing a homogenous good. The total measure of firms is normalized to be 1. All the firms have the same cost c in producing each unit of the good (firms have common cost shocks). Ex ante, c (i.e., the state of the world) can either take value c_H or c_L, where $c_L < c_H$. At the beginning of the period firms observe the realization of the cost and then compete in prices. We also assume that each firm has a capacity constraint k (finite), that is,

[4] For models based on nonsequential search, see, for example, Salop and Stiglitz, 1977; Braverman, 1980; and Varian, 1980. For models based on sequential search, see, for example, Burdett and Judd, 1983; Rob, 1985; and Stahl, 1989.

[5] More specifically, Fishman shows that in the case of a general cost (common to all firms) increase, consumers search too much so as to limit the extent prices increase in the short run; however, in the case of an idiosyncratic cost (specific to only one firm) increase, consumers search too little, leading to price overshooting.

no firm can sell more than k units of the good.[6]

There is a continuum of consumers with total measure $\beta > 1$. The parameter β can also be interpreted as the number of consumers per firm in the market. Each consumer has a unit demand with a choke price of $v > c_H$. We assume that $\beta < k$, that is, the number of consumers per firm in the market is less than each firm's capacity constraint.[7] Consumers do not observe the realization of c. Instead, consumers hold beliefs about the cost realization, which might be heterogenous among consumers. Let α denote a consumer's belief if she believes that the probability of $c = c_H$ is α. The distribution of beliefs among consumers will be specified later. Before observing prices, consumers make decisions regarding whether to search (become informed) or not to search (stay uninformed). We adopt the protocol of nonsequential search. Informed consumers observe all the realized prices and purchase from the firms (stores) with the lowest price available. Each uninformed consumer shops randomly at a firm (store) and only observes that firm's price.

Each consumer's type is characterized by her search cost. The first type of consumers (with proportion λ_1) each have search cost $s_L = 0$. These consumers are also called *shoppers*, who can be interpreted as those who have obtained price information without incurring nontrivial search cost (e.g, from TV or internet advertisements etc.). This type of consumers always search in equilibrium regardless of their beliefs about the underlying state. The second type of consumers (with proportion λ_2) each have search cost s_H. We assume that $s_H > v$ so that this type of consumers never search (regardless of their beliefs about the underlying state). The rest of the consumers (with proportion $1 - \lambda_1 - \lambda_2$) each have intermediate searching cost $s_M \in (s_L, s_H)$ and they may or may not search depending on their beliefs about the underlying state. Since beliefs about the underlying state only matters for this type of consumers, they are henceforth referred to as the *critical consumers*. Let $F(\alpha)$ denote the cumulative distribution function of the beliefs among the critical consumers. That is, let $F(\alpha)$ be the fraction of the critical consumers whose beliefs are lower than α.

Since there is a capacity constraint for each firm, rationing may occur: for a low-price firm, the number of consumers shopping at this firm may be bigger than k. We adopt the proportional rationing rule: if rationing occurs at a firm, each consumer (non-searcher or searcher) who shops at that firm will be able to purchase a unit of the good with the same probability. If a non-searcher

[6] In the dynamic setting, this assumption implies that no firm can sell more than k units of the good per period.

[7] This assumption makes competition among firms non-trivial.

is rationed, she shops randomly at other firms without incurring any cost. If a searcher is rationed, she goes to the firm with the lowest price among the remaining firms. Should rationing also occur there, the same search procedure applies until the searcher purchases a unit of the good.

The timeline of the game is as follows. First, the production cost (the state of the world) is realized and all the firms observe the common state. The firms then simultaneously set prices. Finally, consumers decide whether to search and make purchases accordingly.[8]

As is standard in the search literature, we focus on symmetric equilibria in which all firms employ the same pricing strategies. Let G be the price distribution and μ be the proportion of informed consumers, or the search intensity, which is endogenously determined in our model ($\mu \geq \lambda_1$). A symmetric perfect Bayesian equilibrium is characterized by a pair $(\mu^*, G^*(\cdot|c))$, with the following properties: Given the equilibrium search intensity μ^*, firms' optimal pricing strategies yield the equilibrium price distribution $G^*(\cdot|c)$ given c; Given $G^*(\cdot|c)$, consumers' optimal search decisions give rise to the equilibrium search intensity μ^*.

2.2 Analysis with Fixed Search Intensity

We first derive the equilibrium price distribution given the search intensity μ and the production cost c (the state). Let $\underline{p}$ be the lowest price charged in equilibrium, and let the proportion of the firms that charge $\underline{p}$ be $\eta(\underline{p})$. Note that a $\underline{p}$ firm's sales are:

$$\min\left\{\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta, k\right\}$$

The first term in the bracket is the demand for a $\underline{p}$ firm: it attracts $(1-\mu)\beta$ non-searchers, and gets $\frac{\mu\beta}{\eta(\underline{p})}$ searchers. The quantity a $\underline{p}$ firm sells is simply the minimum of its demand and capacity.

Lemma 1 *In any equilibrium, a firm that charges $\underline{p}$ must sell k units of the good.*

Proof. Suppose in negation, a firm charging $\underline{p}$ sells strictly less than k units in equilibrium. Then by undercutting $\underline{p}$ by an arbitrarily small amount ε, this firm can attract a positive measure of searchers,[9] thus increasing its sales to k without affecting the profit margin, which destroys the

[8] Since the first and the second type consumers either always or never search, only the critical consumers have nontrivial decisions to make.

[9] Since the guarding firms are of measure zero, even if all of them charge lower prices, the deviating firm can still attract a postive measure of searchers.

proposed equilibrium. This implies that in any equilibrium, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \geq k$. ■

Lemma 2 *There is no equilibrium in which prices are continuously distributed on* $[\underline{p}, \overline{p}]$*, for any* $\overline{p}$ *such that* $\underline{p} < \overline{p} \leq v$.

Proof. Consider a candidate equilibrium in which prices are continuously distributed on $[\underline{p}, \overline{p}]$ for some $\overline{p}$ such that $\underline{p} < \overline{p} \leq v$. A necessary condition for this to be an equilibrium is that consumers should be rationed at any $p \in (\underline{p}, \overline{p})$: if consumers are not rationed at such a p, then there is no point to charge $p + \varepsilon$, since a firm can only attract non-searchers in that case and its demand is given by $(1-\mu)\beta$, which is strictly dominated by charging v. But given that consumers are rationed at any $p \in (\underline{p}, \overline{p})$, which means that each firm charging any $p \in (\underline{p}, \overline{p})$ sells k, a $\underline{p}$ firm can increase its profit margin without affecting its sales by deviating to $p \in (\underline{p}, \overline{p})$. This destroys the proposed equilibrium. ■

Lemma 3 *In any equilibrium, consumers shopping at any* $\underline{p}$ *firm are not rationed. That is,* $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \leq k$.

Proof. Suppose in negation, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta > k$. In this case, a positive measure of consumers (hence searchers) are rationed at $\underline{p}$ firms. By Lemma 2, there is a positive number ε such that no firm charges at any price $p \in (\underline{p}, \underline{p} + \varepsilon)$. Then a firm who charges $\underline{p}$ can deviate to some price $p' \in (\underline{p}, \underline{p} + \varepsilon)$. Under this deviation, this firm can still attract enough searchers (since a positive measure of searchers are rationed at $\underline{p}$ firms), and thus sell k units with an increased profit margin. This destroys the proposed equilibrium.

Therefore, $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta \leq k$ in equilibrium, if there is any. This implies that searchers are not rationed at $\underline{p}$ firms. ■

Lemma 1 and Lemma 3 jointly imply that, in equilibrium we must have $\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta = k$. That is, no rationing occurs at firms charging the lowest price. Next we will establish that any equilibrium must have a two-point price distribution.

Lemma 4 *Given* μ *and* c*, in any candidate equilibrium each firm must charge either* $p = v$ *or* $p = \underline{p} \in (c, v)$ *(*$\underline{p}$ *is to be determined).*

Proof. Suppose there is an equilibrium price strictly between $\underline{p}$ and v. Then a firm that charges this price can increase its profit by deviating to charging v. This deviation leads to a higher profit margin per unit of sales, with the demand unchanged (only the lowest price can attract informed consumers, given that they are not rationed at the lowest price by Lemma 3). So the only possible equilibrium is either all firms charging the same price, or a two-price distribution on $\underline{p}$ and v.

First consider the candidate equilibrium in which firms charge the same price $p = \underline{p} > c$. The equilibrium demand for each firm is β. But then a firm can undercut p a little bit and sell $k > \beta$ without affecting the profit margin per unit of sales. Thus this type of equilibrium cannot exist. All firms charging $p = c$ cannot be equilibrium either, because by deviating to $p = v$ a firm can get a positive profit by selling to uninformed consumers ($s_H > v$ means that a measure of $\lambda_2\beta$ consumers are always uninformed).

Now the only candidate equilibrium left is that firms charge either v or $\underline{p}$. Clearly, $\underline{p} < v$. What remains to be shown is that $\underline{p} > c$. Since charging v yields a positive profit, charging $\underline{p}$ should also yield a positive profit, which implies $\underline{p} > c$. ■

By the above lemmas, there is only one possible equilibrium, with equilibrium prices characterized by a two-point distribution. Denote $\pi(v)$ and $\pi(\underline{p})$ as the profits of a firm that charges v and $\underline{p}$, respectively. Explicitly,

$$\begin{aligned} \pi(v) &= (1-\mu)\beta(v-c) \\ \pi(\underline{p}) &= k(\underline{p}-c) \end{aligned}$$

By Lemma 3, a firm charging v can only attract non-searchers. Thus its demand and sales are $(1-\mu)\beta$, and its profit margin is $v-c$.[10] By Lemma 1, a firm charging $\underline{p}$ sells k, with a profit margin $\underline{p}-c$. The equilibrium is characterized by the following two conditions:

$$\pi(v) = \pi(\underline{p}) \tag{1}$$

$$\frac{\mu\beta}{\eta(\underline{p})} + (1-\mu)\beta = k \tag{2}$$

Condition (1) says that a firm should be indifferent between charging v and $\underline{p}$, and condition (2) says that the demand for a $\underline{p}$ firm exactly equals its capacity k.

[10] Note that $(1-\mu)\beta < k$ since $\beta < k$.

Proposition 1 *Given μ and c, there is a unique equilibrium: a proportion of $1-\eta(\underline{p})$ firms charge price v, and a proportion of $\eta(\underline{p})$ firms charge price $\underline{p} \in (c, v)$, where $\eta(\underline{p})$ and $\underline{p}$ are determined by conditions (1) and (2). More explicitly,*

$$\eta(\underline{p}) = \frac{\mu\beta}{k-\beta+\mu\beta} \tag{3}$$

$$\underline{p} = c + \frac{(1-\mu)\beta}{k}(v-c) \tag{4}$$

Proof. We only need to show the price distribution specified above is an equilibrium. Note that from condition (2), searchers are not rationed at $\underline{p}$. We show that firms have no incentive to deviate. First consider a firm charging v. Deviating to any $p \in (\underline{p}, v)$ would lead to a lower profit margin without increasing the sales (because searchers are not rationed at $\underline{p}$), hence such a deviation is not profitable. Deviating to $\underline{p}$ yields the same profit by condition (1). Deviating to $p < \underline{p}$ is strictly dominated by charging $\underline{p}$, since a firm cannot sell more than k. Thus a firm charging v has no incentive to deviate. Next consider a firm charging $\underline{p}$. By a similar argument the firm has no incentive to deviate to $p < \underline{p}$. If the firm deviates to $p \in (\underline{p}, v]$, it only attracts non-searchers and hence sell $(1-\mu)\beta$ only, since searchers are not rationed at $\underline{p}$.[11] Thus the most profitable deviation is to set price at v, which, by condition (1) yields the same profit as no deviation. Thus, a firm charging $\underline{p}$ has no incentive to deviate either. Solving (1) and (2) yields expressions (3) and (4). ■

Note that in this unique equilibrium, no consumer is rationed: the demand for a $\underline{p}$ firm exactly equals its capacity k, and the demand for a v firm is strictly less than k. From (3), we can see that $\eta(\underline{p})$ is increasing in μ, the search intensity. Intuitively, as μ increases the demand becomes more elastic, and more firms charge the lower prices. Moreover, $\eta(\underline{p})$ does not depend on c directly. By (4), $\underline{p}$ is increasing in c and decreasing in μ. As demand becomes more elastic (μ increases), charging the lower price becomes relatively more profitable, other things equal. To restore the indifference condition, the lower price must decrease to reduce the profit margin for the lower price firms. Note

[11] A single firm deviation would not affect the total measure of the firms who set price $\underline{p}$ (due to our assumption of continuum of firms). Thus condition (2) implies the following condition:

$$\frac{\mu\beta}{\text{Measure of firms setting } \underline{p}} + (1-\mu)\beta = k,$$

which in turn implies that searchers will not be rationed, even though one single firm deviates by setting a higher price (all before consumers move, as specified in our timeline).

that the presence of the capacity constraint keeps the profit margin of $\underline{p}$ firms positive, by restricting the competition among $\underline{p}$ firms.

2.3 Equilibrium with Endogenously Determined Search Intensity

Now we analyze the equilibrium of the game, with search intensity μ endogenously determined. We assume that firms know the distribution of beliefs $F(\alpha)$. Recall that the cost c can only take two possible values c_H and c_L, which is the same among all the firms. Proposition 1 characterizes firms' equilibrium price distribution given any search intensity μ. The remaining task is to derive the equilibrium search intensity μ^*. We first derive the expected gain from searching given the equilibrium price distribution, μ, and a consumers' belief α:

$$\begin{aligned} E[p-\underline{p}|\alpha] &= \alpha[(1-\eta(\underline{p}_H))v+\eta(\underline{p}_H)\underline{p}_H-\underline{p}_H]+(1-\alpha)[(1-\eta(\underline{p}_L))v+\eta(\underline{p}_L)\underline{p}_L-\underline{p}_L] \\ &= (1-\eta(\underline{p}_i))[\alpha(v-\underline{p}_H)+(1-\alpha)(v-\underline{p}_L)] \\ &= \frac{k-\beta}{k}\{v-[\alpha c_H+(1-\alpha)c_L]\} \end{aligned} \quad (5)$$

where $\underline{p}_H$ denotes $\underline{p}(c_H)$ and $\underline{p}_L$ denotes $\underline{p}(c_L)$. Note that by (4), $\underline{p}_H > \underline{p}_L$ given μ.

From (5) we can see that $E[p-\underline{p}|\alpha]$ does not depend on μ. Thus search does not exhibit complementarity. This is because an increase in μ has two countervailing effects. The first effect is that $\underline{p}$ decreases as μ increases, which increases the return of search. On the other hand, an increase in μ causes more firms to charge $\underline{p}$ ($\eta(\underline{p})$ increases), which reduces the average price and raises the payoff of a non-searcher. These two effects exactly offset each other, as shown by (5).

From (5), we see that the expected gain from search is decreasing in α for critical consumers. Thus there is a cutoff belief $\widehat{\alpha}$ such that all critical consumers with beliefs below $\widehat{\alpha}$ search and those above $\widehat{\alpha}$ do not search. If $s_M \geq \frac{k-\beta}{k}(v-c_L)$, even the consumer with the most optimistic belief ($\alpha=0$) cannot afford the search so $\widehat{\alpha}=0$; on the other hand, if $s_M \leq \frac{k-\beta}{k}(v-c_H)$, even the consumer with the most pessimistic belief ($\alpha=1$) can afford the search so $\widehat{\alpha}=1$. In what follows we will focus on the most interesting case in which

$$\frac{k-\beta}{k}(v-c_H) < s_M < \frac{k-\beta}{k}(v-c_L) \quad (6)$$

In this case, $\widehat{\alpha}$ is interior to $(0,1)$, and is determined by the following indifference condition:

$$s_M = E[p-\underline{p}|\widehat{\alpha}] = \frac{k-\beta}{k}\{v-[\widehat{\alpha}c_H+(1-\widehat{\alpha})c_L]\}. \quad (7)$$

Solving (7) we have

$$\widehat{\alpha} = \frac{(v - c_L)(k - \beta) - ks_M}{(c_H - c_L)(k - \beta)} \tag{8}$$

Then the equilibrium search intensity μ^* can be computed as follows.

$$\mu^* = \lambda_1 + (1 - \lambda_1 - \lambda_2)F(\widehat{\alpha}) \tag{9}$$

Note that condition (7) pins down a unique $\widehat{\alpha} \in [0, 1]$, and hence μ^* is also unique. It is easily seen that μ^* and $G^*(\mu^*)$ described by (3) and (4) constitute the unique equilibrium of the static game. Given $G^*(\mu^*)$, the optimal decision rules about search give rise to μ^*. Since firms know $F(\alpha)$, they can correctly anticipate μ^* by (9). And given μ^*, the firms' optimal pricing strategies result in the equilibrium price distribution $G^*(\mu^*)$. Note that rationing does not occur in equilibrium. This is the case for two reasons. First, firms can correctly anticipate the equilibrium search intensity μ^*. Second, although $\underline{p}_H > \underline{p}_L$, $\eta(\underline{p}_H) = \eta(\underline{p}_L)$, that is, the proportion of the firms charging the lower price does not depend on the cost realization.

The equilibrium price distribution is determined by the cost realization and consumers' beliefs $F(\alpha)$. The average price is lower under state c_L than under state c_H. Moreover, the price distribution is more dispersed under state c_L (the gap between the average price and the lowest price is larger), which leads to a higher expected return to search. Thus as more consumers' beliefs lie below $\widehat{\alpha}$ (or $F(\widehat{\alpha})$ increases), the equilibrium search intensity μ^* increases. As a result, both $\underline{p}_H$ and $\underline{p}_L$ are decreasing in $F(\widehat{\alpha})$, and $\eta(\underline{p}_H) = \eta(\underline{p}_L)$ are increasing in $F(\widehat{\alpha})$. Define the average equilibrium price as $\overline{p}\ (c_i, \mu^*)$:

$$\begin{aligned}\overline{p}(c_i, \mu^*) &= \eta(\underline{p}_i)\underline{p}_i + [1 - \eta(\underline{p}_i)]v \\ &= \frac{\mu^*\beta}{k - \beta + \mu^*\beta}[c + \frac{(1 - \mu^*)\beta}{k}(v - c)] + \frac{k - \beta}{k - \beta + \mu^*\beta}v \end{aligned} \tag{10}$$

It is easily seen from (10) that $\overline{p}(c_i, \mu^*)$ is increasing in c_i and decreasing in μ^*. Thus the average price is also decreasing in $F(\widehat{\alpha})$. The following proposition summarizes these results.

Proposition 2 *There is a unique equilibrium with the equilibrium search intensity given by (8) and (9). Moreover, μ^* is increasing in $F(\widehat{\alpha})$; both $\underline{p}_H$ and $\underline{p}_L$ are decreasing in $F(\widehat{\alpha})$, and both $\eta(\underline{p}_H)$ and $\eta(\underline{p}_L)$ are increasing in $F(\widehat{\alpha})$; $\overline{p}(c_i, \mu^*)$ is decreasing in μ^* and $F(\widehat{\alpha})$.*

According to the previous analysis, changes in equilibrium price distribution can be decomposed into two components. The first component is the change resulting from changes in cost realization, and the second component is the change resulting from changes in consumers' search intensity, which is govern by the distribution of consumers' beliefs. This decomposition will be illustrated when we turn to the dynamic model.

3 Dynamic Model

We now extend our analysis to the dynamic setting, and endogenize critical consumers' beliefs. Time t is discrete and $t = 1, 2, ...$ In each period the static game is played. We assume that the common cost evolves according to a Markov process, with ρ being the persistence parameter. That is,

$$\Pr(c_{t+1} = c_H | c_t = c_H) = \Pr(c_{t+1} = c_L | c_t = c_L) = \rho$$

where $\rho > 1/2$. At $t = 1$, the two cost states are equally likely. This Markov structure of cost evolution is common knowledge. While firms always observe the cost state of the current period, consumers never observe past or current cost realizations. Instead, each consumer updates her belief about the cost based on the price history she observes. Tappata (2006) assumes that last period cost realization is observable to consumers, thus prices adjust fully to cost shocks in two periods.[12]

From the static model we see that the equilibrium price distribution in a particular period depends on consumers' beliefs and the cost realization. The key in our dynamic game analysis is to trace the belief updating process among critical consumers. From the static model, the lower price $\underline{p}$ is responsive to cost realizations: $\underline{p}_H > \underline{p}_L$ for any given μ. To simplify our analysis, we make assumptions about the parameter values such that the lower bound of $\underline{p}_H$ (under the highest possible μ) is greater than the upper bound of $\underline{p}_L$ (under the lowest possible μ). Note that the highest possible μ is $1 - \lambda_2$ (all critical consumers search), and the lowest possible μ is λ_1 (no critical consumers searches). More specifically, we assume

$$\frac{c_H - c_L}{v - c_L} > \frac{\beta(1 - \lambda_1 - \lambda_2)}{k - \beta\lambda_2} \tag{11}$$

which implies $\underline{p}_H(\mu = 1 - \lambda_2) > \underline{p}_L(\mu = \lambda_1)$, i.e., there is no overlaps between the supports of $\underline{p}_H$ and $\underline{p}_L$ in equilibrium (hence (11) can also be termed as the *non-overlapping condition*).

[12] Since the past cost history is assumed to be known to the consumers, there is no learning in Tappata's model.

Given this non-overlapping condition, a consumer who observes $\underline{p}$ can correctly infer the true cost of that period. As a result, her initial belief next period is either ρ or $1-\rho$. On the other hand, the high price (v) is not responsive to cost realizations. Thus if a consumer observes a price v her belief about the true cost in the current period (α_t^p, superscript p denotes posterior) is updated as follows (suppose her initial belief is that $\Pr(c_t = c_H) = \alpha_t$).

$$\alpha_t^p = \frac{\alpha_t(1-\eta(\underline{p}_H))}{\alpha_t(1-\eta(\underline{p}_H)) + (1-\alpha_t)(1-\eta(\underline{p}_L))} = \alpha_t$$

since by (3) $\eta(\underline{p}_H) = \eta(\underline{p}_L)$ in any period. Hence the consumer's belief is not updated at all. As a result, her initial belief about the cost for the next period will be:

$$\alpha_{t+1} = \rho\alpha_t + (1-\rho)(1-\alpha_t) \tag{12}$$

Note that a consumer's initial belief has an upper bound ρ and a lower bound $1-\rho$. Moreover, if a critical consumer with initial belief ρ observes v in all the subsequent n periods, then by (12) her initial belief converges to 1/2 from above. To further facilitate our analysis, in what follows we will assume that s_M has bounds tighter than given in (6). That is,

$$\frac{k-\beta}{k}[(v-c_L) - \frac{1}{2}(c_H - c_L)] < s_M < \frac{k-\beta}{k}[(v-c_L) - (1-\rho)(c_H - c_L)] \tag{13}$$

Under this assumption, it can be verified that $\widehat{\alpha} \in (1-\rho, 1/2)$. That $\widehat{\alpha} > 1-\rho$ ensures that critical consumers search under the most optimistic belief; otherwise they will not search regardless of their beliefs. That $\widehat{\alpha} < 1/2$ ensures that critical consumers with initial belief ρ who observe v in all the subsequent periods will not search. Though not affecting the robustness of our result, assumption (13) does simplify our analysis as we do not need to trace the beliefs of those consumers who do not search.

From the above discussion, we can see that searchers and non-searchers have different belief updating processes. Since a searcher always observes the low price, she can infer the true cost state correctly. However, a non-searcher only observes one price, and this price may be the high price v. In this case her posterior is not updated. If she observes the lower price, she updates her posterior correctly. As we will see, the difference in belief updating processes between searchers and non-searchers is the key to understanding asymmetric price adjustments.

3.1 Equilibrium

By equation (8), $\widehat{\alpha}$ does not depend on μ_t^* (the equilibrium search intensity in period t) or the cost state in period t. Therefore, the distribution of beliefs in period t can be summarized by $F_t(\widehat{\alpha})$. Hence in each period, the state of the economy can be summarized by $F_t(\widehat{\alpha})$ and c_t, and the equilibrium in the dynamic game can be characterized by the sequence of $\{F_t(\widehat{\alpha}), c_t, \mu_t^*, G^*(\cdot|c_t)\}$.

Proposition 3 *The equilibrium in the dynamic game exists and is unique for any evolution of* $\{c_t\}$.

Proof. Suppose firms correctly anticipates $F_t(\widehat{\alpha})$. Then the equilibrium μ_t^* is determined by

$$\mu_t^* = \lambda_1 + (1 - \lambda_1 - \lambda_2)F_t(\widehat{\alpha}) \tag{14}$$

From the analysis of the static game, the equilibrium price distribution in period t, $G^*(\cdot|\mu_t^*, c_t)$ is uniquely determined. The existence of equilibrium in the dynamic game boils down to the following conditions: firms are able to anticipate $F_t(\widehat{\alpha})$ correctly, and the uniqueness is guaranteed if the evolution of $F_t(\widehat{\alpha})$ is unique given c_t. Below we will show that these two properties hold.

First, we show that firms can infer $F_1(\widehat{\alpha})$. In period $t = 1$ all critical consumers hold the same initial belief $1/2$ (since there is no prior history of prices). Given $\widehat{\alpha} < 1/2$, $F_1(\widehat{\alpha}) = 0$, and no critical consumers search in period $t = 1$. Note that since it is common knowledge that all critical consumers hold the same initial belief $1/2$ in period 1, firms can correctly infer that $F_1(\widehat{\alpha}) = 0$.

Second, we show that a critical consumer's belief will never be in $(\widehat{\alpha}, 1/2)$. To see this, first note that each critical consumer's initial belief in period 1 is $1/2$. If a critical consumer observes a sequence of higher prices v, her initial belief remains $1/2$ by (12). Her belief will be different only if she observes the lower price in the last period. In that case, her initial belief will either be ρ or $1 - \rho$. If her belief is $1 - \rho$, then she will search and observe the low price, and her next period belief will be either ρ or $1 - \rho$. If her belief is ρ, then it will either converge to $1/2$ from above if she keeps observing the high price, or it will be revised to ρ or $1 - \rho$ if she happens to observe the low price in some period. Therefore, in all cases a critical consumer's belief is outside the range of $(\widehat{\alpha}, 1/2)$.

Third, we show that if the firms know $F_t(\widehat{\alpha})$, then combining the information about c_t, they can correctly infer $F_{t+1}(\widehat{\alpha})$. We discuss two cases in order.

In the first case, suppose $c_t = c_H$. For critical consumers whose $\alpha_t \leq \widehat{\alpha}$, they will search in period t and learn the true state $c_t = c_H$; hence their initial belief in period $t+1$ is ρ. For critical consumers whose $\alpha_t \in [1/2, \rho]$, they will not search in period t. Among those consumers, an $\eta_t(\underline{p}_H)$ portion observe $\underline{p}_H$ and learn the true state c_H. Thus their initial belief in period $t+1$ becomes ρ. The remaining $1-\eta_t(\underline{p}_H)$ portion of consumers observes v hence no updating occurs. Their initial belief in period $t+1$ remains to be within $[1/2, \rho]$. Aggregating over all the critical consumers, we can see that $F_{t+1}(\widehat{\alpha}) = 0$. Thus we have the following transition equation for the distribution of beliefs:

$$\text{If } c_t = c_H\text{: } F_{t+1}(\widehat{\alpha}) = 0 \text{ gvien any } F_t(\widehat{\alpha}) \tag{15}$$

In the second case, suppose $c_t = c_L$. For critical consumers whose $\alpha_t \leq \widehat{\alpha}$, they will search in period t and learn the true state $c_t = c_L$; hence their initial belief in period $t+1$ is $1-\rho$. For critical consumers whose $\alpha_t \in [1/2, \rho]$, they will not search in period t. Among those consumers, an $\eta_t(\underline{p}_L)$ portion observe $\underline{p}_L$ and learn the true state c_L. Thus their initial belief in period $t+1$ becomes $1-\rho$. The remaining $1-\eta_t(\underline{p}_H)$ portion of consumers observe v with no information updated. Their initial belief in period $t+1$ remains to be in $[1/2, \rho]$. Aggregating over all the critical consumers, we obtain another transition equation for the distribution of beliefs:

$$\begin{aligned} \text{If } c_t &= c_L\text{: } F_{t+1}(\widehat{\alpha}) = F_t(\widehat{\alpha}) + [1 - F_t(\widehat{\alpha})]\eta_t(\underline{p}_L) \\ &= F_t(\widehat{\alpha}) + \frac{\mu_t^* \beta}{k - \beta + \mu_t^* \beta}[1 - F_t(\widehat{\alpha})] \end{aligned} \tag{16}$$

From (15) and (16), we can see that given c_t and $F_t(\widehat{\alpha})$, $F_{t+1}(\widehat{\alpha})$ is uniquely determined. Moreover, firms can correctly anticipate $F_{t+1}(\widehat{\alpha})$ based on their information about c_t, $F_t(\widehat{\alpha})$ and μ_t^*.

Therefore, given the evolution of $\{c_t\}$, there is a unique equilibrium characterized by $\{F_t(\widehat{\alpha}), c_t, \mu_t^*, G^*(\cdot|c_t)\}$ in the dynamic game. ■

Note that except for period 1, critical consumers' beliefs are heterogenous due to different price histories they experienced. Note that we do not need to trace the evolution of the exact distribution of beliefs $F_t(\alpha)$, which would be very cumbersome to describe. Instead, we only need to trace the evolution of $F_t(\widehat{\alpha})$ to determine the equilibrium search intensity μ_t^*. Another interesting property is that, while firms can correctly infer $F_t(\widehat{\alpha})$, μ_t^* and $G^*(\cdot|c_t)$, consumers in period t do not need to hold the correct beliefs about $F_t(\widehat{\alpha})$, μ_t^* and $G^*(\cdot|c_t)$. The main reason is that consumers do not observe the history $\{c_t\}$; Instead, they update their beliefs about $\{c_t\}$ based on their personal histories of the prices they encountered.

3.2 Equilibrium Properties

According to the analysis of the static game, changes in equilibrium price distribution can be decomposed into two changes, one due to the change in cost realization, and the other due to the change in consumers' search intensity. In the dynamic game, the highest average price (and the highest lower price) arise when $c_t = c_H$ and $\mu_t^* = \underline{\mu} = \lambda_1$ (no critical consumers searches). Note that this corresponds to the case where $c_t = c_H$ and consumers have full information about c_t. On the other hand, the lowest average price (and the lowest lower price) arise when $c = c_L$ and $\mu_t^* = \overline{\mu} = 1 - \lambda_2$ (all critical consumers search). Similarly, this corresponds to the case where $c = c_L$ and consumers have full information about c_t. Therefore, in state c_H we say price is fully adjusted if $\mu^* = \underline{\mu} = \lambda_1$, and in state c_L we say that price is fully adjusted if $\mu^* = \overline{\mu} = 1 - \lambda_2$.

In the dynamic game, since consumers do not observe the past cost realizations, their beliefs hence the search intensity μ^* do not adjust as quickly as the underlying cost state changes. More importantly, the speed of adjustments for the beliefs and search intensity are different under positive cost shocks and negative cost shocks. Since the average price and the lower price move in the same direction, for brevity of exposition in what follows they are often simply referred to as *the prices*. The following propositions identify the asymmetry in price adjustments.

Proposition 4 *Suppose a positive cost shock occurs in period $t+1$. Then regardless of $F_{t+1}(\widehat{\alpha})$, $F_{t+2}(\widehat{\alpha}) = 0$ and $\mu_{t+2}^* = \underline{\mu} = \lambda_1$. Regardless of previous history, if cost states LHH are realized in periods $t, t+1$, and $t+2$, then the prices fully adjust to the highest level in period $t+2$.*

Proof. Suppose L and H are the realized cost states for period t and $t+1$ respectively. Then by the transition equation (15), $F_{t+2}(\widehat{\alpha}) = 0$ irrespective of $F_{t+1}(\widehat{\alpha})$. By (14), $\mu_{t+2}^* = \lambda_1 = \underline{\mu}$. Therefore, if $c_{t+2} = c_H$, the prices reach the highest level, hence are fully adjusted in period $t+2$. ■

Proposition 4 implies that state H is the absorbing state in terms of critical consumers' search behavior: regardless of the previous history, if a positive shock occurs in the current period, the search intensity will fully adjust downward in the next period. Another implication of Proposition 4 is that the prices fully adjust upward in two periods when a positive cost shock occurs and the cost stays high in the next period. More specifically, in the first period a positive shock occurs, prices only adjust upward partially since the search intensity is not fully adjusted. However, in the second period the search intensity fully adjusts downward, leading to fully upward adjustment of prices.

The key to this result is that, when a positive shock occurs, the critical consumers who search in the current period immediately learn the cost state has switched to H, thus they stop searching in the next period. For the non-searchers, they either observe the lower price, learn the true state H and do not search in the next period, or observe the high price, do not update their beliefs and remain to be non-searchers in the next period.

Proposition 5 *Suppose a negative cost shock occurs in period $t+1$. Then*

$$F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k - \beta + \underline{\mu}\beta} < 1 \text{ and } \mu_{t+2}^{*} < \overline{\mu}.$$

Suppose the L cost state persists in the subsequent n periods after period $t+1$. Then $F_{t+1+n}(\widehat{\alpha})$ increases in n, and converges to 1 as n goes to infinity. Hence the prices are not fully adjusted downward in period $t+2$. Instead, the prices decrease gradually and converge to the lowest possible price as n goes to infinity.

Proof. Suppose H and L are the realized cost states for period t and $t+1$, respectively. According to Proposition 4, $F_{t+1}(\widehat{\alpha}) = 0$, and $\mu_{t+1}^{*} = \lambda_1 = \underline{\mu}$. By the transition equation (16), $F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k-\beta+\underline{\mu}\beta}$, which is clearly less than 1. If the L state persists in the subsequent n periods, by the transition equation (16), $F_{t+1+n}(\widehat{\alpha})$ will be strictly increasing in n, and eventually converges to 1 as n approaches infinity. Accordingly, the search intensity will gradually adjust upward and the prices will gradually adjust downward. The adjustment process will be completed only when n goes to infinity. ■

Proposition 5 implies that when a negative cost shock occurs, the downward price adjustment is a gradual process and takes long time to complete. The underlying reason is that in our model, when a negative shock occurs, no critical consumer searches initially (recall that H is an absorbing state). Thus only those consumers who happen to observe the lower price learn the true state (L) and begin searching in the next period. For consumers who observe the high price, they do not learn the true state is L, and remain to be non-searchers in the next period. As a result, the prices do not fully adjust to the lowest level within two periods. If the L cost state persists in the subsequent n periods, more and more non-searchers observe the lower price, learn the L state, and begin to search. Actually, the rate at which the measure of non-searchers decreases in period t is the proportion of firms who set the lower price, $\frac{\mu_t^{*}\beta}{k-\beta+\mu_t^{*}\beta}$, by the transition equation (16).

Asymmetric price adjustments can be seen from comparing Propositions 4 and 5: when a positive cost shock occurs, prices fully adjust upward in two periods, while when a negative cost shock occurs, it takes much longer periods for prices to fully adjust downward. To see the magnitude of the asymmetry, we evaluate the magnitude of the adjustment in the search intensity in the first two periods after a negative cost occurs. By Proposition 5, within two periods $F(\widehat{\alpha})$ is adjusted to $\frac{\underline{\mu}\beta}{k-\beta+\underline{\mu}\beta}$. Since $\underline{\mu}$ is relatively small, this amount of adjustment is small compared to the full adjustment level, which is 1. Therefore, the adjustments of the search intensity in the first two periods are relatively small, which implies that the adjustments of prices in the first two periods are also relatively small and a significant portion of the price adjustment is completed in later periods.

Now we examine the asymmetric price adjustments by focusing on some specific cost evolution paths. Given the random nature of underlying state switches, it would be impossible to trace all the possible evolution paths of the underlying states; Instead we will work with the *expected evolution path* of the underlying states. Given the persistence parameter ρ, in expectation each given state (H or L) will remain for $\frac{1}{1-\rho} \equiv N$ consecutive periods and before switching to the other state ($N > 2$ since $\rho > 1/2$). We thus focus on the following expected evolution path of the states: $L...LH...HL...L...$. That is, N periods of state L followed by N periods of state H, which are in turn followed by N periods of state L, and so on.

Along this expected evolution path, the lowest prices will occur in the Nth period of the L state, and the highest prices emerge in the 2nd through the Nth periods of the H state. The pattern of asymmetric price adjustment is quite remarkable. When a positive cost shock occurs, the prices adjust from the lowest (the Nth period of the L state) to the highest within two periods (the 2nd period of the H state), and then the prices remain at the same level until the Nth period of the H state. On the other hand, when a negative cost shock occurs, it takes $N > 2$ periods for the prices to adjust from the highest (the Nth period of the H state) to the lowest (the Nth period of L state), before the state switches to H. During the N periods of the L state, the price decreases gradually to the lowest level. We summarize the result below.

Proposition 6 *Along the expected evolution path, price adjustments exhibit asymmetric pattern in equilibrium. While the prices always fully adjusted upward within two periods after a positive cost shock occurs, it takes longer time for the prices to fully adjust downward in case of negative cost shocks.*

Numerical examples will be provided in subsection 3.4 below to demonstrate this pattern of asymmetric adjustments. The underlying reason for asymmetric adjustments lies in the asymmetric belief updating, which results from the consumers' search behavior. While searchers always learn the true cost state immediately, non-searchers do not learn the true cost state at all unless they are lucky to observe the lower price. When there is a positive cost shock, the searchers among critical consumers immediately learn that the cost has gone up. As a result, those consumers stop searching in the next period and prices are fully adjusted upward. On the other hand, when the cost goes down, those consumers who observe the high price do not learn the true state and remain as non-searchers, and only those who observe the lower price learn the true state and begin to search. As a result, the aggregated beliefs of the consumers is gradually adjusted downward as more and more non-searchers observe the lower price, which leads to gradual downward price adjustments.

3.3 Comparative Statics

In this subsection we study how changes in some exogenous parameters affect the pattern of price adjustments. First, we provide a measure to evaluate the degree of asymmetry in price adjustments. Denote the magnitude of upward (downward) price adjustment within two periods after a positive (negative) cost shock occurs as MA^P (MA^N). We define the adjustment ratio AR as follows.

$$AR = \frac{MA^N}{MA^P}$$

AR measures the degree of asymmetry of the price adjustments: the smaller the AR, the smaller the magnitude of downward price adjustment within two periods relative to that of upward price adjustment within two periods, hence the more asymmetric the price adjustments.

We start with the changes in the persistence parameter ρ. Consider two Markov processes 1 and 2, with $\rho_2 > \rho_1 > 1/2$, that is, Markov process 2 is more persistent than process 1. In the previous analysis about the expected evolution path, we see that N increases as ρ increases. Thus we have $N_2 > N_1$. Note that $\underline{\mu} = \lambda_1$ does not depend on ρ. Hence the highest prices on the two paths are the same. However, since $N_2 > N_1$, the lowest prices under process 2 (the N_2th period in state L) is lower than those under process 1 (the N_1th period in state L). As a result, $MA_1^P < MA_2^P$. On the other hand, MA^N does not depend on ρ, which is evident from the equation

$$F_{t+2}(\widehat{\alpha}) = \frac{\underline{\mu}\beta}{k - \beta + \underline{\mu}\beta} = \frac{\lambda_1\beta}{k - \beta + \lambda_1\beta}. \tag{17}$$

Thus $MA_1^N = MA_2^N$. As a result, $AR_1 > AR_2$. That is, the asymmetric price adjustments are more prominent when the cost evolution is more persistent.

Another way to understand this result is as follows. As the Markov process becomes more persistent, the magnitude of full price adjustments becomes larger. In the case of positive shocks, this larger magnitude of upward price adjustment is still completed within two periods. However, in the case of negative shocks the downward price adjustment is spreading over longer periods. This makes the price adjustment more asymmetric. The following proposition summarizes the result.

Proposition 7 *As the Markov process becomes more persistent (ρ increases), the asymmetric pattern of price adjustments becomes more prominent.*

Note that Proposition 7 is a testable implication.

Now consider the impact of λ_1 (the proportion of shoppers) on the pattern of price adjustment. Let $\lambda_1' < \lambda_1$. We show that the downward price adjustment in the case of a negative shock is slower under λ_1' than under λ_1.

Proposition 8 *As λ_1 decreases, when a negative cost shock occurs the prices adjust downward more slowly.*

Proof. Suppose a negative shock occurs in period $t+1$, and the L state persists in the subsequent periods. Our goal is to show that the prices in any period $t+j$ ($j \geq 2$) are strictly lower under λ_1 than under λ_1'. Since prices are decreasing in the search intensity, it is sufficient to show that $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$ for all $j \geq 2$ (superscript plier is used to distinguish variables under parameter λ_1' from those under λ_1). According to equation (17), $F_{t+2}'(\widehat{\alpha}) < F_{t+2}(\widehat{\alpha})$. Now we proceed with induction. In the first step we show that if $F_{t+j}'(\widehat{\alpha}) < F_{t+j}(\widehat{\alpha})$, then $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$ for $j \geq 2$. The second step proves that if $F_{t+j}'(\widehat{\alpha}) < F_{t+j}(\widehat{\alpha})$ and $\mu_{t+j}^{*\prime} < \mu_{t+j}^{*}$ then $F_{t+j+1}'(\widehat{\alpha}) < F_{t+j+1}(\widehat{\alpha})$.

The first step. By equation (14),

$$\begin{aligned}
\mu_{t+j}^{*} - \mu_{t+j}^{*\prime} &= (\lambda_1 - \lambda_1') + (1 - \lambda_1 - \lambda_2)F_{t+j}(\widehat{\alpha}) - (1 - \lambda_1' - \lambda_2)F_{t+j}'(\widehat{\alpha}) \\
&= (\lambda_1 - \lambda_1')[1 - F_{t+j}'(\widehat{\alpha})] + (1 - \lambda_1 - \lambda_2)[F_{t+j}(\widehat{\alpha}) - F_{t+j}'(\widehat{\alpha})] \\
&> 0.
\end{aligned}$$

The second step. First, define

$$A_{t+j} \equiv \frac{\mu_{t+j}^{*}\beta}{k - \beta + \mu_{t+j}^{*}\beta}.$$

$A_{t+j} > A'_{t+j}$ since $\mu^*_{t+j} > \mu^{*\prime}_{t+j}$, and both of them are strictly between 0 and 1. Now by the transition equation (16),

$$\begin{aligned} F_{t+j+1}(\widehat{\alpha}) - F'_{t+j+1}(\widehat{\alpha}) &= [F_{t+j}(\widehat{\alpha}) - F'_{t+j}(\widehat{\alpha})] + A_{t+j}[1 - F_{t+j}(\widehat{\alpha})] - A'_{t+j}[1 - F_{t+j}(\widehat{\alpha})] \\ &= (A_{t+j} - A'_{t+j})[1 - F_{t+j}(\widehat{\alpha})] + [F_{t+j}(\widehat{\alpha}) - F'_{t+j}(\widehat{\alpha})](1 - A'_{t+j}) \\ &> 0 \end{aligned}$$

Thus we obtain the desired result. ■

Intuitively, when the proportion of shoppers is smaller, in the case of a negative cost shock fewer consumers search initially. As a result, fewer proportion of firms set the lower price. This reduces the speed of learning for the critical consumers, leading to slower adjustments of the search intensity and the prices.

Though a smaller λ_1 slows down the process of downward price adjustment, the upward price adjustment is not affected: it is still completed within two periods. Thus a decrease in λ_1 makes the pattern of asymmetric price adjustments more prominent.

By a similar argument, we can show that an increase in k/β leads to a slower downward price adjustment. The intuition is that an increase in k/β, other things equal, results in fewer firms setting the lower price. This slows down the critical consumers' learning speed when a negative cost shock occurs, leading to slower downward price adjustment.

3.4 Numerical Examples

We now provide numerical examples to illustrate how asymmetric price adjustments work in our model. We choose the following parameter values:

$$\beta = 3, k = 5, \lambda_1 = 0.1, \lambda_2 = 0.1, c_H = 0.6, c_L = 0.2, v = 0.8, s_M = 0.2.$$

It is easily verified that under this set of parameters, the non-overlapping condition (11) is satisfied. We will focus on the expected cost evolution path by examining two cases, $\rho = 0.8$ and $\rho = 0.9$ in order.[13]

In the first case, the expected duration for a given state is 5 periods. Without loss of generality we assume that such an expected path begins with state H. Therefore a negative shock occurs at

[13] It is easily verified that condition (13) is satisfied under these parameters.

time $t = 6$. After 5 periods of state L, a positive shock occurs at time $t = 11$. We compute and report the equilibrium search intensity, the equilibrium fraction of firms setting the low price, the equilibrium low price and average price in the following table.

$\rho = 0.8$

t	State	μ_t^*	η_t	$\underline{p}_t$	Avg. price
5	H	.1000	.1304	.7080	.7880
6	L	.1000	.1304	.5240	.7640
7	L	.2043	.2346	.4864	.7264
8	L	.3675	.3554	.4277	.6677
9	L	.5568	.4551	.3595	.5995
10	L	.7130	.5168	.3033	.5433
11	H	.8096	.5484	.6228	.7028
12	H	.1000	.1304	.7080	.7880
13	H	.1000	.1304	.7080	.7880

It is clear from the table that the price adjustment exhibits asymmetric pattern. At $t = 5$, since the previous state is H, no critical consumers search so the search intensity is $\lambda_1 = .1$. The state switches to L at $t = 6$, but the search intensity remains the same since it is still the case that no critical consumers search. As a result, the fraction of firms setting low price is not adjusted (which remains to be 13.04%), and the equilibrium average price only drops from .7880 to .7640. In the following period ($t = 7$), the search intensity increases to .2043. The fraction of firms setting low price increases to .2346, and the average price further decreases to .7264. This process continues until the average price reaches the lowest level, .5433, at $t = 10$. Note that this declining process is quite slow. For example, during the first two periods of the adjustment, the reduction of the average price only accounts for 25.17% of the total adjustment from $t = 5$ through $t = 10$. However, the adjustment pattern is very different when a positive cost shock occurs, since it takes only two periods for the price to be fully adjusted: When the state switches to H at $t = 11$, the search intensity remains to be high (at .8096), and the average price increases from .5433 to .7028. In the following period, the search intensity drops all the way to the lowest possible level (.1000), and the average price is fully adjusted to the highest possible level (.7880). So the adjustment process in response to a positive cost shock is completed in only two periods, which is much faster than the adjustment process in response to a negative cost shock.

We next consider the case with $\rho = 0.9$. Again we follow the expected evolution path where each state persists for 10 periods (a negative shock occurs at $t = 11$and a positive shock occurs at $t = 21$ in this case). The results are reported in the following table.

$\rho = 0.9$

t	State	μ_t^*	η_t	$\underline{p}_t$	Avg. price
10	H	.1000	.1304	.7080	.7880
11	L	.1000	.1304	.5240	.7640
12	L	.2043	.2346	.4864	.7264
13	L	.3675	.3554	.4277	.6677
14	L	.5568	.4551	.3595	.5995
15	L	.7130	.5168	.3033	.5433
16	L	.8096	.5484	.2685	.5085
17	L	.8592	.5631	.2507	.4907
18	L	.8822	.5696	.2424	.4824
19	L	.8923	.5724	.2388	.4788
20	L	.8967	.5736	.2372	.4772
21	H	.8986	.5741	.6122	.6922
22	H	.1000	.1304	.7080	.7880
23	H	.1000	.1304	.7080	.7880

Again we see that the adjustment in response to the positive cost shock is completed in two periods, while the adjustment in response to the negative cost shock is much slower. With the state evolution process being more persistent than in the above example, now the pattern of asymmetric adjustments is more striking: only 19.82% of the average price adjustment is competed within the first two periods under a negative cost shock, while full adjustment is again completed in two periods for a positive cost shock.

3.5 Discussions

Our model is stylized, as we made some simplifying assumptions to facilitate our analysis. Here we discuss why we impose these assumptions and whether they affect the general insight of the paper.

Continuum of firms with capacity constraint Unlike the standard search literature, which considers a finite number of firms, we work with an infinite number of firms and each firm is subject to a capacity constraint. These two specific assumptions are adopted for technical convenience. Working with a finite number of firms without capacity constraint would involve two technical difficulties. The first difficulty is that the equilibrium of the static game (with endogenously determined search intensity) cannot be analytically derived, as shown in Tappata (2006) with more than three firms.[14] The second difficulty with a finite number of firms is that the inference about the underlying state would be too complicated. With a finite number of firms, each firm chooses a price according to some distribution (usually with a continuous distribution within a compact support). Thus the Bayesian updating based on different realized prices can easily get very involved. Moreover, finite number of firms adds randomness in the realized prices in each period, which further complicates the analysis of belief updating. With a continuum of firms and capacity constraints in our model, we pin down a simple two-point distribution in equilibrium. Moreover, the evolution of consumers' beliefs (hence equilibrium search intensity) is deterministic given the evolution of the underlying states, though individual firms might play mixed strategy in setting prices. This greatly simplifies the analysis and makes the Bayesian inference tractable.

We believe that the price dispersions in the real world markets should be, in general, represented by some complicated price distribution functions. We do not attempt to argue that the two-point price distribution derived in our model is most reasonable; Instead, we only want to argue that the two-point price distribution is tractable for our analysis, while capturing the essence of the real world price dispersions.

We believe that the general insight of our model also carries over to the setting with a finite number of firms. In the static game, consumers have stronger incentives to search when they believe that the low cost state is more likely, since in a low cost state prices are more dispersed due to competition among firms. In the dynamic setting, asymmetry in learning is still present. Searchers observe all the prices set by the firms, while each non-searcher only observes one price. As a result, the searchers learn the true state quicker than non-searchers. This would naturally lead to asymmetric price adjustments. Such insight, we believe, should also carry over to the models with finite number of firms.

[14] The main reason is that the expected gain of search cannot be integrated out explicitly.

$\underline{p}_H$ and $\underline{p}_L$ do not overlap In the dynamic game, we make the assumption (11) such that the equilibrium ranges of $\underline{p}_H$ and $\underline{p}_L$ do not overlap. As a result, whenever a non-searcher observes the lower price, she learns the true cost state immediately. In a more general model, we should allow for the case that $\underline{p}_H$ and $\underline{p}_L$ may overlap in equilibrium. Under such a setting, we believe that the general insight of the paper still holds. To see this, note that searchers can always infer the true cost state immediately. They have two pieces of information, the lower price $\underline{p}$ and the proportion of firms charging the lower price $\eta(\underline{p})$. From two equations (3) and (4), they can infer the two unknowns, μ and c, correctly. However, the belief updating for non-searchers would be much more involved. If a non-searcher observes a lower price which lies in the overlapping range, she cannot immediately infers the true state. While it is difficult to pin down the exact belief updating process in this case,[15] it is clear that non-searchers' belief updating is slower than that of the searchers. Given this, asymmetric price adjustment will still emerge: when a positive shock occurs, searchers stop searching immediately as they immediately learn the cost is high; when a negative cost shock occurs, it takes much longer time for non-searchers to learn that the state has switched to low and they should start searching.

4 Conclusion

In this paper we build a simple search model with learning to demonstrate how asymmetric price adjustments can arise as firms' optimal responses to cost shocks. While the upward price adjustment is always completed within two periods after a positive cost shock occurs, the downward price adjustment takes much longer to complete when a negative shock occurs.

The underlying reason for asymmetric price adjustments is that searchers and non-searchers have different belief updating processes. Since searchers observe the whole spectrum of price distribution while non-searchers only observe one single price, searchers learn the true cost state a lot quicker than non-searchers do. This learning asymmetry naturally leads to asymmetric price adjustments. When a positive cost shock occurs, searchers quickly learn the true state and stop searching. Thus the quick downward adjustment of search intensity leads to the quick upward price adjustment. On the other

[15] We need to derive the distribution of the lower price in the dynamic game (which is presumably a daunting job), and then apply Bayesian rule to pin down the belief updating process.

hand, when a negative cost shock occurs, it takes a much longer period of time for non-searchers to learn the true state and start searching. This slow upward adjustment of search intensity leads to slow downward price adjustment.

Thus our paper provides an explanation for the widespread phenomenon of asymmetric price adjustments. While our model is simple, we believe that it captures the essence of the real world product markets with imperfectly informed consumers. Our model also predicts that asymmetric price adjustments are more prominent in markets where the cost shocks are more persistent or where more critical consumers are present. These are testable empirical implications.

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